FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND APPLIED MATHEMATICS

MODULE	ASMA1B1 APPLICATIONS OF CALCULUS
CAMPUS	APK
ASSESSMENT	JUNE EXAM

DATE 28/05/2019	TIME 12:30
ASSESSOR(S)	S RICHARDSON
INTERNAL MODERATOR	DR A SWARTZ
DURATION 120 MINUTES	MARKS 70

SURNAME AND INITIALS _____

STUDENT NUMBER _____

CONTACT NUMBER _____

NUMBER OF PAGES: 1 + 12 PAGES

INSTRUCTIONS: 1. ANSWER ALL THE QUESTIONS ON THE PAPER IN PEN.
2. NO CALCULATORS ARE ALLOWED.
3. SHOW ALL CALCULATIONS AND MOTIVATE ALL ANSWERS.
4. IF YOU REQUIRE EXTRA SPACE, CONTINUE ON THE ADJACENT BLANK PAGE AND INDICATE THIS CLEARLY.

Question 1 [10 marks]

For questions 1.1 - 1.10, choose **one** correct answer, and make a cross (X) in the correct block.

Question	**a**	**b**	**c**	**d**	**e**
1.1					
1.2					
1.3					
1.4					
1.5					
1.6					
1.7					
1.8					
1.9					
1.10					

1.1 Consider the integral $\int \frac{x^3}{\sqrt{16-x^2}}\,dx$. Which of the following is the correct trigonometric substitution? (1)

a) $x = 4\sin\theta,\ \frac{-\pi}{2} < \theta < \frac{\pi}{2}$

b) $4x = \sin\theta,\ \frac{-\pi}{2} \leqslant \theta \leqslant \frac{\pi}{2}$

c) $4x = \sin\theta,\ \frac{-\pi}{2} < \theta < \frac{\pi}{2}$

d) $x = 4\sin\theta,\ \frac{-\pi}{2} \leqslant \theta \leqslant \frac{\pi}{2}$

e) None of the above

1.2 If $y = x^{-2}\ln x$ and $\frac{dy}{dx} = \frac{1-2\ln x}{x^3}$ then the critical number(s) of the function y are: (1)

a) 0 only

b) 0 and $\sqrt{e}$

c) 0 and e^2

d) $\sqrt{e}$ only

e) None of the above

1.3 $\int_0^1 \sqrt{x^2-2x+1}\,dx = ?$ (1)

a) $\frac{1}{2}$

b) 1

c) $-\frac{1}{2}$

d) divergent

e) None of the above.

1.4 The area of the region bounded by $y=\sqrt{x},\ y=0$ and $x=9$ is (1)

a) 27

b) $\frac{81}{2}$

c) 16

d) 18

e) None of the above

1.5 The length of the path described by the parametric equation $x=\frac{1}{3}t^3$ and $y=\frac{1}{2}t^2$ for $0\le t\le 1$, is given by : (1)

a) $\int_0^1 \sqrt{t^2+1}\,dt$

b) $\int_0^1 \sqrt{t^4+t^2}\,dt$

c) $\frac{1}{2}\int_0^1 \sqrt{4+t^4}\,dt$

d) $\int_0^1 \sqrt{t^2+t}\,dt$

e) None of the above

1.6 Two numbers whose difference is 50 and whose product is a minimum are: (1)

a) 25 and -25
b) 30 and -20
c) 70 and 20
d) 25 and 25
e) None of the above

1.7 The point on the curve $y = \sqrt{x}$ that is closest to the point (2,0) is: (1)

a) $(\sqrt{\frac{3}{2}}, \frac{3}{2})$

b) $(\frac{3}{2}, \sqrt{\frac{3}{2}})$

c) $(\sqrt{\frac{5}{2}}, \frac{5}{2})$

d) $(\sqrt{\frac{1}{2}}, \frac{1}{2})$

e) None of the above

1.8 Suppose $y = \sqrt{3x+1}$ where x and y are functions of t. If $\frac{dx}{dt} = 10$, find $\frac{dy}{dt}$ when $x = 8$. (1)

a) $\frac{dy}{dt} = 1$

b) $\frac{dy}{dt} = 5$

c) $\frac{dy}{dt} = 3$

d) $\frac{dy}{dt} = 10$

e) None of the above

1.9 $\sum_{k=0}^{n} \binom{n}{k} 9^k = \ldots$ (1)

a) 10^n

b) 9^n

c) 9^k

d) 10^k

e) None of the above

1.10 If two equations have no values to satisfy both equations, then this is called a (1)

a) consistent system
b) inconsistent system
c) solution system
d) constant system
e) None of the above

Question 2 [7 marks]

(a) State Rolle's Theorem (2)

(b) Give the three cases that are considered when proving Rolle's Theorem (3)

(c) State Fermat's Theorem (2)

Question 3 [14 marks]

Sketch the graph $f(x) = \dfrac{x^2}{x^2+4}$. In the preliminary analysis, complete the following:

(a) Domain (1)

(b) Intercepts (2)

(c) Asymptotes (2)

(d) Intervals of increasing and decreasing (3)

(e) Local maximum and minimum values (1)

(f) Concavity (3)

(g) Sketch $f(x)$ (2)

Question 4 [9 marks]

(a) Find the area of the region bounded by the curves: $2y^2 = x + 2$, $x = y^2$. (4)

(b) Use the disk or washer method to find the volume of the solid obtained by rotating the region bounded by $y = 1 - x^2$ and $y = 0$ about the line $y = 2$. (5)

Question 5 [6 marks]

Evaluate each of the following integrals if they exist.

(a) $\int_{\frac{\pi}{2}}^{\pi} \cos^5 x \, dx$ (3)

(b) $\int_1^e \frac{dx}{x \ln x}$ (3)

Question 6 [5 marks]

When a bullet is fired into a sand bank, its retardation is equal to the square root of its velocity v, thus $\frac{dv}{dt} = -\sqrt{v}$, where t is the time in seconds after the bullet has entered the sand bank.

(a) Find a formula to determine the velocity at any time t, by solving the given differential equation. (2)

(b) If the velocity on entering the sand bank is $49m/s$, how long does the bullet travel before it stops completely? (3)

Question 7 [4 marks]

Solve the differential equation for $y(0) = 1$: $\quad y' - \dfrac{2y}{x+1} = (x+1)^{5/2}$

Question 8 [4 marks]

Find all points on $x = 4\cos t, \quad y = 4\sin t$ that have a slope of $\dfrac{1}{\sqrt{3}}$

Question 9 [2 marks]

Set up, but do not evaluate, an integral for the area under the parametric curve given by :

$$x = 2a\cos t - a\cos 2t; \quad y = 2a\sin t - a\sin 2t; \quad 0 \leq t \leq 2\pi$$

Question 10 [6 marks]

(a) Find a system of linear equations corresponding to the given augmented matrix: (2)

$$\begin{bmatrix} 3 & 0 & -2 & 5 \\ 7 & 1 & 4 & -3 \\ 0 & -2 & 1 & 7 \end{bmatrix}$$

(b) Find the augmented matrix for the given system of linear equations: (1)

$$x_1 - x_4 = 7$$
$$x_2 + x_4 = -1$$

(c) Solve the sytsem in (b). (3)

Question 11 [3 marks]

(a) Expand to the first three terms of the expresion $(y - 2x)^5$. Simplify all coefficients. (2)

(b) **Hence**, find the coefficient of x^3y^4 in the expression $(1 + x + 2x^2)(y - 2x)^5$. (1)