

Department of Pure and Applied Mathematics Differential Equations A (APM8X10) – Exam

28 May, 2019

Examiner: Prof. Fabio Cinti

Duration: 3h

Total marks: 60

This is a closed-book examination

Note: numbers in brackets [] indicate the marks that are awarded for each of the four questions if correctly solved.

1. Consider the initial-value problem

$$x^2 y'' - xy' + y = x^2, \quad y(1) = 4, \quad y'(1) = 3.$$

Solve the equation using the Green's functions formalism.

[15]

2. Use the power series method to solve the given initial-value problem

$$y'' - 2xy' + 8y = 0, \quad y(0) = 3, \quad y'(0) = 0.$$

[15]

3. Consider the non-homogenous linear system

$$\begin{cases} \frac{dx_1(t)}{dt} = x_1(t) + x_2(t) + e^t \\ \frac{dx_2(t)}{dt} = x_1(t) + x_2(t) + e^{2t} \\ \frac{dx_3(t)}{dt} = x_3(t) + te^{3t}. \end{cases}$$

Solve the given system using the variation of parameters method.

[15]

4. Consider the autonomous system

$$\begin{cases} x' = x(1 - x^2 - 3y^2) \\ y' = y(3 - x^2 - 3y^2). \end{cases}$$

Finds all critical points.

[15]