UNIVERSITY OF JOHANNESBURG

Department of Economics and Econometrics

Faculty of Economic and Financial Sciences

FINAL ASSESSMENT: JUNE 2019

Course	:	Econometrics 4A (EKN4814, ECM8X01)
Moderator	:	Dr A Pholo
Date	:	3 June 2019
Time	:	180 minutes
Marks	:	100 points

Question 1 (5 pts)

Consider an alternative formulation of the Cobweb model called *adaptative expectations*. Let the expected price in t (denoted by p_t^*) be a weighted average of the price in $t-1$ and the price expectation of the previous period. Formally,

$$p_t^* = \alpha p_{t-1} + (1-\alpha)\, p_{t-1}^* \qquad 0 < \alpha \leq 1 \tag{1}$$

An interesting feature of this model is that it can be viewed as a difference equation expressing the expected price as a function of its own lagged value and the forcing variable p_{t-1}.

1. Find the homogeneous solution p_t^*.

2. Use lag operators to find the particular solution. Check your answer by substituting your answer into the original difference equation.

Question 2 (10 pts)

Find the particular solution of the following equation $y_t = a_1 y_{t-1} + \epsilon_t$ by the method of undetermined coefficients.

Question 3 (10 pts)

Part 1 (5 pts)

For each of the following, determine whether $\{y_t\}$ represents a stable process. Determine whether the characteristic roots are real or imaginary and whether the real parts are positive or negative.

1. $y_t - 1.2y_{t-1} + 0.2y_{t-2}$

2. $y_t - 1.2y_{t-1} + 0.4y_{t-2}$

3. $y_t - 1.2y_{t-1} - 1.2y_{t-2}$

4. $y_t + 1.2y_{t-1}$

5. $y_t - 0.7y_{t-1} - 0.25y_{t-2} + 0.175y_{t-3}$ [Hint: $(x - 0.5)(x + 0.5)(x - 0.7) = x^3 - 0.7x^2 - 0.25x + 0.175$]

Part 2 (5 pts)

Write each of the above equations using lag operators. Determine the characteristic roots of the inverse characteristic equation.

Question 4 (10 pts)

Use the Yule-Walker equations to find the autocorrelation function of an ARMA$(1,1)$ process.

Question 5 (5 pts)

Under which conditions an AR(1) process is covariance-stationary? Prove that if those conditions are fullfilled the process meets all the conditions of covariance-stationarity.

Question 6 (10 pts)

Regarding the Box-Jenkins approach:

1. Explain the concept of parsimony in the Box-Jenkins approach for the estimation of ARMA series. Let us consider the two following models $y_t = 0.4y_{t-1} + \varepsilon_t$ and $y_t = \epsilon_t + 0.4\epsilon_{t-1} + 0.16\epsilon_{t-2} + 0.064\epsilon_{t-3} + 0.0256\epsilon_{t-4}$. Which one would you prefer to estimate? Explain the common factor problem. How to handle it.

2. Explain the concept of stationarity and invertibility.

Question 7 (10 pts)

The file QUARTERLY.XLS contains the U.S. money supply as measured by M1 (M1NSA) and as measured by M2 (M2NSA). The series are quarterly average over the period 1960Q1 to 2008Q2.

1. Using M1 and assuming that $m_t = (1-L)\left(1-L^4\right) y_t$ with $y_t = log\left(M1NSA_t\right)$, estimate the following three models and select the best:

   - $m_t = a_0 + a_1 m_{t-1} + \varepsilon_t + \beta_4 \varepsilon_{t-4}$ Model 1: AR(1) with Seasonal MA

- $m_t = a_0 + (1 + a_1 L)\left(1 + a_4 L^4\right) m_{t-1} + \varepsilon_t$ Model 2: Multiplicative AR

- $m_t = a_0 + (1 + \beta_1 L)\left(1 + \beta_4 L^4\right) \varepsilon_t$ Model 3: Multiplicative MA

2. Compare those 3 models to a model with a seasonal $AR(1)$ term with an additive $MA(1)$ term.

3. Obtain the ACF for the growth rate of the M2NSA series. What type of model is suggested by the ACF?

4. Call the seasonally differenced growth rate $m2_t$. Estimate an $AR(1)$ model with a seasonal MA term over the 1962Q3 to 2008Q2 period. Show that this model is preferable to (i) an $AR(1)$ model with a seasonal AR term, (ii) $MA(1)$ with a seasonal AR term, and (iii) an $MA(1)$ with a seasonal MA term.

5. What would you recommend to remove any serial correlation in the residuals?

Question 8 (5 pts)

Define the random walk plus noise model by its mathematical expression. Describe and explain the different trend and stationary components of this model. What are the extensions of this model?

Question 9 (10 pts)

To illustrate the use of the various test statistics, Dickey and Fuller (1981) used quarterly values of the logarithm of the Federal Reserve Board's Production Index over the 1950Q1-1977Q4 period to estimate the following three equations:

$$
\begin{aligned}
\Delta y_t &= \underset{(0.15)}{0.52} + \underset{(0.00034)}{0.00120t} - \underset{(0.033)}{0.119 y_{t-1}} + \underset{(0.081)}{0.498 \Delta y_{t-1}} + \varepsilon_t \quad SSR = 0.056448 \\
\Delta y_t &= \underset{(0.0025)}{0.0054} + \underset{(0.083)}{0.447 \Delta y_{t-1}} + \varepsilon_t \quad SSR = 0.063211 \\
\Delta y_t &= \underset{(0.079)}{0.511 \Delta y_{t-1}} + \varepsilon_t \quad SSR = 0.065966
\end{aligned}
$$

where SSR = sum of squared residuals, and standard errors are in parentheses. Using the τ_τ statistic and the table of the empirical cumulative distribution of τ test whether the y_t series is stationary. Perform also the ϕ_2 and ϕ_3 tests. Explain the results.

Question 10 (10 pts)

Considering the following primitive bivariate VAR system:

$$
y_t = b_{10} - b_{12} z_t + \gamma_{11} y_{t-1} + \gamma_{12} z_{t-1} + \epsilon_{yt} \tag{2}
$$

$$
z_t = b_{20} - b_{21} y_t + \gamma_{21} y_{t-1} + \gamma_{22} z_{t-1} + \epsilon_{zt} \tag{3}
$$

where it is assumed that both y_t and z_t are stationary; ϵ_{yt} and ϵ_{zt} are white-noise disturbances with standard deviations of σ_y and σ_z, respectively; ϵ_{yt} and ϵ_{zt} are uncorrelated white-noise disturbances. Derive the corresponding VAR system in standard form. What are the properties of the errors terms of this standard form system?

Question 11 (15 pts)

The data set MONEY_DEM.XLS contains real U.S. GDP (RGDP), nominal GDP, the money supply as measured by M2, and the three-month rate on U.S. Treasury bills. Construct the following four variables:

$$
\begin{aligned}
\text{dlrgdp}_t &= \ln(RGDP_t) - \ln(RGDP_{t-1}) \\
\text{price}_t &= GDP_t / RGDP_t \\
\text{dlrm2}_t &= \ln(M2_t/\text{price}_t) - \ln\left(M2_{t-1}/\text{price}_{t-1}\right) \\
\text{drs}_t &= \text{tb3mo}_t - \text{tb3mo}_{t-1}
\end{aligned}
$$

1. Estimate a three-variable VAR with 12 lags of dlrgdp_t, dlrm2_t, and drs_t. Include a constant but do not use any seasonal dummy variables.

2. Calculate the multivariate AIC and SBC, using the following formulas $\text{AIC} = T\ln|\Sigma| + 2N$ and $\text{SBC} = T\ln|\Sigma| + N\ln(T)$.

3. Reestimate the model using eight lags for each variable. Compare the VAR models with respectively 12 and 8 endogenous variables, using Sims χ^2 test and the multivariate AIC and SBC.

4. Using the 12-lag model, check whether drs_t is block exogenous for the other two variables in the system.

5. Perform the Granger causality tests.