UNIVERSITY OF JOHANNESBURG

FACULTY/COLLEGE	College of Business and Economics
SCHOOL	School of Economics
CAMPUS(ES)	APK
MODULE NAME	Econometrics 2A
MODULE CODE	ECM2A02
SEMESTER	First
ASSESSMENT OPPORTUNITY, MONTH AND YEAR	Final Summative Assessment June/July/August 2019

ASSESSOR(S)	Ms Bonolo Thobejane and Ms Lebo Nleya		
MODERATOR(S)	Dr. Marinda Pretorius		
DURATION	3 hours	**TOTAL MARKS**	120

NUMBER OF PAGES OF QUESTION PAPER (Including cover page And formula sheet)	**13**

INSTRUCTIONS:

- This is a closed-book assessment.
- Read the questions carefully and answer only what is required.
- Number your answers clearly and correctly as per the question paper.
- Write neatly and legibly on both sides of the paper in the answer book, starting on the first page.
- Round off all final answers to four decimal points unless otherwise stated

Section A: Multiple Choice **[20 Marks]**

Choose the correct answer in the questions below:

1. If selected items are not returned to a population, the binomial probability distribution can be used to approximate the hypergeometric distribution when:
   a. $n<0.5N$
   b. $n\leq0.5N$
   c. $n<0.05N$
   d. $n\leq0.05N$
2. Examples of measures of position include(s):
   a. Median
   b. Mean
   c. Percentiles
   d. (a)(b)(c)
3. Which of the following is not true about the type of data that can be used to calculate the median?
   a. Nominal level data
   b. Interval level data
   c. Ordinal level data
   d. (a)(b)(c)
4. Chebyshev's theorem postulates that, for any set of observations, the proportion of the values that lie within k standard deviations of the mean is at least $1-1/k^2$, where k is any value greater than:
   a. 0
   b. 0.5
   c. 1
   d. 1.5
5. Which of the following is true about values that probability can assume?
   a. 0.5
   b. 1
   c. -0.5
   d. (a)(b)
6. For a population that is not normally distributed, the distribution of the sample means will:
   a. Be negatively skewed
   b. Approach the normal distribution
   c. Be positively skewed
   d. Take the same shape as the population

7. Decreasing the sample size, while holding the confidence level the same, will do what to the length of your confidence interval?
   a. Increase
   b. Decrease

c. Stay the same
d. Cannot be determined from the given information

8. A result is called "statistically significant" whenever:
   a. The null hypothesis is true
   b. The alternative hypothesis is true
   c. The p-value is less or equal to the significance level
   d. The p-value is larger than the significance level
9. In hypothesis testing, a Type II error occurs when:
   a. The null hypothesis is not rejected when the null hypothesis is true
   b. The null hypothesis is rejected when the null hypothesis is true
   c. The null hypothesis is not rejected when the alternative hypothesis is true
   d. The null hypothesis is rejected when the alternative hypothesis is true
10. Typically one-way ANOVA is used in which of the following situations?

    I. There are several distinct populations
    II. There are two sample populations over 4000
    III. Randomized experiments
    IV. Randomly selected populations

    a. All of the above
    b. II and III only
    c. I, II, and III only
    d. I, and III only

Section B: True/False **[20 Marks]**

Determine whether the following statements are true or false:

1. For data that is recorded at ordinal level of measurement, the distance between the values is meaningful.

2. A coefficient of skewness ranges between -3 and +3.

3. A frequency table is a grouping of qualitative data into mutually exclusive classes showing the number of observations in each class.

4. To compute the mean, the data must quantitative and qualitative.

5. For empirical probability, the probability of an event happening is the fraction of the time similar events happened in the past.

6. A statement about a population developed for the purpose of testing is called hypothesis testing.

7. When systematic random sampling is used, the central limit theorem cannot be applied.

8. Fish and game wardens estimate the average weight of the fish or game population by using creel checks and other devices. Based on this sample data, a warden might estimate that the mean weight of Coho salmon caught in Lake Michigan is 2.5 pounds. This single number is called a point estimate of the unknown population parameter.

9. The standard error of the mean is also called the sampling error.

10. ANOVA is the preferred method for finding differences among several population proportions.

Section C: Theory **[20 marks]**

1. Explain the four major characteristics of the standard deviation statistic. (8)

2. Define a probability distribution. (2)

3. Complete the table below by providing the correct term for the definitions. Provide your answer in the answer sheet. (10)

a. The standard deviation of the sample mean	
b. A single value used to estimate a population parameter	
c. A range of values within which the population parameter is expected to occur	
d. Not rejecting the null hypothesis when it is true	
e. A value calculated from sample information used to determine whether to reject the null hypothesis	
f. Assuming the null hypothesis is true, the likelihood of finding a value of the test statistic at least as extreme as the one found in the sample	
g. a population is divided into subgroups called strata and a sample is randomly selected from each stratum	
h. The variable that is being predicted in regression analysis	
i. A second treatment variable that when included in the ANOVA analysis will have the effect of reducing the SSE term	
j. A mathematical procedure that uses the data to position a line with the objective of minimizing the sum of the squares of the vertical distances between the actual y values and the predicted values of y	

Section D: Calculations **[60 Marks]**

1. The annual imports of a selected group of electronic suppliers are shown in the frequency distribution (*see also ECM2A02 excel sheet 1*):

Imports ($millions)	Number of suppliers
2 up to 5	6
5 up to 8	13
8 up to 11	20
11 up to 14	10
14 up to 17	1

Brian Tau would like you to assist him in extending the frequency distribution above, by including two columns that show relative frequency and cumulative frequency. Report the results in your answer sheet. (10)

2. The Thomas Supply Inc. is a distributor of gas-powered generators. As with any business, the length of time customers take to pay their invoices is important. Listed below is the time in days for a sample of The Thomas Supply Company Inc. invoices (see also *ECM2A02 excel sheet 2)*:

13	13	13	20	26	27	31	34	34	34	35	35	36	37	38
41	41	41	45	47	47	47	50	51	53	54	56	62	67	82

   a. Determine the first and second quartile (4)
   b. Determine the second decile and eighth decile (4)
   c. Determine the 67th percentile (2)

3. A federal study reported that 7.5% of the US workforce has a drug problem. A drug enforcement official for the state of Indiana wished to investigate this statement. In her sample of 20 employed workers:
   a. How many would you expect to have a drug problem? What is the standard deviation? (4)
   b. What is the likelihood that none of the workers sampled have a drug problem? (2)
   c. What is the likelihood that at least one has a drug problem? (2)
   d. What is the likelihood that at most five have a drug problem? (2)

4. Puma's annual report states that the average South African buys 6.5 pairs of sports shoes per year. Suppose a sample of 81 customers is surveyed and the population standard deviation of sports shoes purchased per year is 2.1.

a. What is the standard error of the mean in this experiment? (2)
b. What is the probability that the sample mean is between 6 and 7 pairs of sports shoes? (3)

5. A random sample of married people were asked "Would you remarry your spouse if you were given the opportunity for a second time?". Of the 150 people

surveyed, 127 of them said that they would do so. Find a 95% confidence interval for the proportion of married people who would remarry their spouse. (4)

6. Suppose that we wanted to estimate the true average number of eggs a queen bee lays with 95% confidence. The margin of error we are willing to accept is 0.5. Suppose we also know that standard deviation is about 10. What sample size should we use? (2)

7. The mean life of a battery used in a digital clock is 305 days. The lives of the batteries follow the normal distribution. The battery was recently modified to last longer. A sample of 20 of the modified batteries had a mean life of 311 days with a standard deviation of 12 days. Did the modification increase the mean life of the battery? Use the hypothesis testing procedure and the 0.05 significance level. (5)

8. Use the data provided in *ECM2A02 excel sheet 3* and consider the following: The City of Joburg comprises of four districts. Chief of police Bheki Cele wants to determine whether there is a difference in the mean number of crimes committed among the four districts. He examined the records from six randomly selected days and recorded the number of crimes. At the 0.05 significance level, can Bheki Cele conclude that that there is a difference in the mean number of crimes among the four districts? (5)

9. Use the data provided in *ECM2A02 excel sheet 4* and consider the following:
   a. What is the strength of the linear relationship between the 2 specified variables? (2)
   b. Calculate and report the equation of the regression line. (3)
   c. Report and interpret the determination coefficient. (2)
   d. What will the estimated value of the ***y*** be if ***x*** is 100 in the next period? (2)

FORMULA SHEET

GEOMETRIC MEAN $GM = \sqrt[n]{(X_1)(X_2)\cdots(X_n)}$ [3–4]

POPULATION VARIANCE $\sigma^2 = \frac{\Sigma(X-\mu)^2}{N}$ [3–8]

ARITHMETIC MEAN OF GROUPED DATA $\bar{X} = \frac{\Sigma fM}{n}$ [3–12]

where:
- $\bar{X}$ is the designation for the sample mean.
- M is the midpoint of each class.
- f is the frequency in each class.
- fM is the frequency in each class times the midpoint of the class.
- ΣfM is the sum of these products.
- n is the total number of frequencies.

STANDARD DEVIATION, GROUPED DATA $s = \sqrt{\frac{\Sigma f(M-\bar{X})^2}{n-1}}$ [3–13]

where:
- s is the symbol for the sample standard deviation.
- M is the midpoint of the class.
- f is the class frequency.
- n is the number of observations in the sample.
- $\bar{X}$ is the designation for the sample mean.

Location of a percentile: $L_p = (n+1)\frac{P}{100}$

P (A or B) = P (A) + P (B)

P (A or B) = P (A) + P (B) – P (A and B) when A and B are not mutually exclusive events

P (A) + P (~A) = 1

P (A and B) = P (A) * P (B)

$$P(X) = \frac{1}{b-a}$$

$$\mu = \frac{a+b}{2}$$

$$\sigma^2 = \frac{(b-a)^2}{12}$$

GENERAL RULE OF MULTIPLICATION $P(A \text{ and } B) = P(A)P(B|A)$ [5-6]

BAYES' THEOREM $P(A_i|B) = \frac{P(A_i)P(B|A_i)}{P(A_1)P(B|A_1) + P(A_2)P(B|A_2)}$ [5–7]

MULTIPLICATION FORMULA Total number of arrangements $= (m)(n)$ [5–8]

PERMUTATION FORMULA ${}_nP_r = \frac{n!}{(n-r)!}$ [5–9]

where:
- n is the total number of objects.
- r is the number of objects selected.

$$\text{Combination}: \text{nCr} = \frac{n!}{r!(n-r)!}$$

MEAN OF A PROBABILITY DISTRIBUTION	$\mu = \Sigma[xP(x)]$	[6–1]
VARIANCE OF A PROBABILITY DISTRIBUTION	$\sigma^2 = \Sigma[(x - \mu)^2 P(x)]$	[6–2]
BINOMIAL PROBABILITY FORMULA	$P(x) = {}_nC_x\, \pi^x (1 - \pi)^{n-x}$	[6–3]

where:

C denotes a combination.
n is the number of trials.
x is the random variable defined as the number of successes.
π is the probability of a success on each trial.

MEAN OF A BINOMIAL DISTRIBUTION	$\mu = n\pi$	[6–4]
VARIANCE OF A BINOMIAL DISTRIBUTION	$\sigma^2 = n\pi(1 - \pi)$	[6–5]
HYPERGEOMETRIC DISTRIBUTION	$P(x) = \dfrac{({}_SC_x)({}_{N-S}C_{n-x})}{{}_NC_n}$	[6–6]

where:

N is the size of the population.
S is the number of successes in the population.
x is the number of successes in the sample. It may be 0, 1, 2, 3, ...
n is the size of the sample or the number of trials.
C is the symbol for a combination.

POISSON DISTRIBUTION	$P(x) = \dfrac{\mu^x e^{-\mu}}{x!}$	[6–7]

where:

μ (mu) is the mean number of occurrences (successes) in a particular interval.
e is the constant 2.71828 (base of the Napierian logarithmic system).
x is the number of occurrences (successes).
$P(x)$ is the probability for a specified value of x.

$$t-value=\frac{\bar{x}-\mu}{s/\sqrt{n}} \qquad z-value=\frac{\bar{x}-\mu}{\sigma/\sqrt{n}}, \qquad n=\left(\frac{z_{\alpha/2}\times\sigma}{E}\right)^2, \qquad z=\frac{\bar{x}-\mu}{\sigma}, \qquad \bar{x}\pm z_{\alpha/2}\times\frac{\sigma}{\sqrt{n}},$$

$$\bar{x}\pm t_{\alpha/2,n-1}\times\frac{s}{\sqrt{n}}$$

Degrees of freedom $= n_1 + n_2 - 2 \quad OR \quad df = \dfrac{\left[\left(s_1^2/n_1\right)+\left(s_2^2/n_2\right)\right]^2}{\frac{(s_1^2/n_1)^2}{n_1-1}+\frac{(s_2^2/n_2)^2}{n_2-1}}$

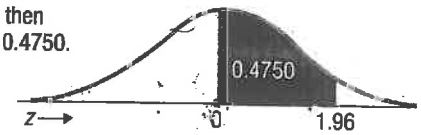
$$Z=\frac{P_1-P_2}{\sqrt{\frac{P_1(1-P_1)}{n_1}+\frac{P_2(1-P_2)}{n_2}}} \quad \text{OR} \quad z=\frac{\bar{X}_s-\bar{X}_u}{\sqrt{\frac{\sigma_s^2}{n_s}+\frac{\sigma_u^2}{n_u}}}$$

$$t=\frac{\bar{X}_1-\bar{X}_2}{\sqrt{s_p^2*\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} \quad \text{OR} \quad t=\frac{\bar{X}_1-\bar{X}_2}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}}$$

$$t=\frac{\bar{d}}{s_d/\sqrt{n}} \qquad s_p^2=\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}$$

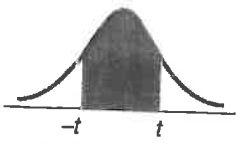
B.3 Areas under the Normal Curve

Example:
If $z = 1.96$, then
$P(0 \text{ to } z) = 0.4750$.

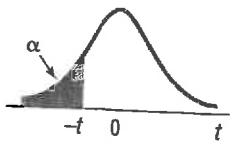


z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

B.5 Student's *t* Distribution

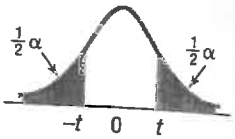


Confidence interval



Left-tailed test

Right-tailed test



Two-tailed test

	Confidence Intervals, *c*					
	80%	90%	95%	98%	99%	99.9%
df	Level of Significance for One-Tailed Test, α					
	0.10	0.05	0.025	0.01	0.005	0.0005
	Level of Significance for Two-Tailed Test, α					
	0.20	0.10	0.05	0.02	0.01	0.001
1	3.078	6.314	12.706	31.821	63.657	636.619
2	1.886	2.920	4.303	6.965	9.925	31.599
3	1.638	2.353	3.182	4.541	5.841	12.924
4	1.533	2.132	2.776	3.747	4.604	8.610
5	1.476	2.015	2.571	3.365	4.032	6.869
6	1.440	1.943	2.447	3.143	3.707	5.959
7	1.415	1.895	2.365	2.998	3.499	5.408
8	1.397	1.860	2.306	2.896	3.355	5.041
9	1.383	1.833	2.262	2.821	3.250	4.781
10	1.372	1.812	2.228	2.764	3.169	4.587
11	1.363	1.796	2.201	2.718	3.106	4.437
12	1.356	1.782	2.179	2.681	3.055	4.318
13	1.350	1.771	2.160	2.650	3.012	4.221
14	1.345	1.761	2.145	2.624	2.977	4.140
15	1.341	1.753	2.131	2.602	2.947	4.073
16	1.337	1.746	2.120	2.583	2.921	4.015
17	1.333	1.740	2.110	2.567	2.898	3.965
18	1.330	1.734	2.101	2.552	2.878	3.922
19	1.328	1.729	2.093	2.539	2.861	3.883
20	1.325	1.725	2.086	2.528	2.845	3.850
21	1.323	1.721	2.080	2.518	2.831	3.819
22	1.321	1.717	2.074	2.508	2.819	3.792
23	1.319	1.714	2.069	2.500	2.807	3.768
24	1.318	1.711	2.064	2.492	2.797	3.745
25	1.316	1.708	2.060	2.485	2.787	3.725
26	1.315	1.706	2.056	2.479	2.779	3.707
27	1.314	1.703	2.052	2.473	2.771	3.690
28	1.313	1.701	2.048	2.467	2.763	3.674
29	1.311	1.699	2.045	2.462	2.756	3.659
30	1.310	1.697	2.042	2.457	2.750	3.646
31	1.309	1.696	2.040	2.453	2.744	3.633
32	1.309	1.694	2.037	2.449	2.738	3.622
33	1.308	1.692	2.035	2.445	2.733	3.611
34	1.307	1.691	2.032	2.441	2.728	3.601
35	1.306	1.690	2.030	2.438	2.724	3.591
36	1.306	1.688	2.028	2.434	2.719	3.582
37	1.305	1.687	2.026	2.431	2.715	3.574
38	1.304	1.686	2.024	2.429	2.712	3.566
39	1.304	1.685	2.023	2.426	2.708	3.558
40	1.303	1.684	2.021	2.423	2.704	3.551
41	1.303	1.683	2.020	2.421	2.701	3.544
42	1.302	1.682	2.018	2.418	2.698	3.538
43	1.302	1.681	2.017	2.416	2.695	3.532
44	1.301	1.680	2.015	2.414	2.692	3.526
45	1.301	1.679	2.014	2.412	2.690	3.520
46	1.300	1.679	2.013	2.410	2.687	3.515
47	1.300	1.678	2.012	2.408	2.685	3.510
48	1.299	1.677	2.011	2.407	2.682	3.505
49	1.299	1.677	2.010	2.405	2.680	3.500
50	1.299	1.676	2.009	2.403	2.678	3.496
51	1.298	1.675	2.008	2.402	2.676	3.492
52	1.298	1.675	2.007	2.400	2.674	3.488
53	1.298	1.674	2.006	2.399	2.672	3.484
54	1.297	1.674	2.005	2.397	2.670	3.480
55	1.297	1.673	2.004	2.396	2.668	3.476
56	1.297	1.673	2.003	2.395	2.667	3.473
57	1.297	1.672	2.002	2.394	2.665	3.470
58	1.296	1.672	2.002	2.392	2.663	3.466
59	1.296	1.671	2.001	2.391	2.662	3.463
60	1.296	1.671	2.000	2.390	2.660	3.460
61	1.296	1.670	2.000	2.389	2.659	3.457
62	1.295	1.670	1.999	2.388	2.657	3.454
63	1.295	1.669	1.998	2.387	2.656	3.452
64	1.295	1.669	1.998	2.386	2.655	3.449
65	1.295	1.669	1.997	2.385	2.654	3.447
66	1.295	1.668	1.997	2.384	2.652	3.444
67	1.294	1.668	1.996	2.383	2.651	3.442
68	1.294	1.668	1.995	2.382	2.650	3.439
69	1.294	1.667	1.995	2.382	2.649	3.437
70	1.294	1.667	1.994	2.381	2.648	3.435

(continued)

B.5 Student's *t* Distribution (*concluded*)

	Confidence Intervals, *c*					
	80%	90%	95%	98%	99%	99.9%
	Level of Significance for One-Tailed Test, α					
df	0.10	0.05	0.025	0.01	0.005	0.0005
	Level of Significance for Two-Tailed Test, α					
	0.20	0.10	0.05	0.02	0.01	0.001
71	1.294	1.667	1.994	2.380	2.647	3.433
72	1.293	1.666	1.993	2.379	2.646	3.431
73	1.293	1.666	1.993	2.379	2.645	3.429
74	1.293	1.666	1.993	2.378	2.644	3.427
75	1.293	1.665	1.992	2.377	2.643	3.425
76	1.293	1.665	1.992	2.376	2.642	3.423
77	1.293	1.665	1.991	2.376	2.641	3.421
78	1.292	1.665	1.991	2.375	2.640	3.420
79	1.292	1.664	1.990	2.374	2.640	3.418
80	1.292	1.664	1.990	2.374	2.639	3.416
81	1.292	1.664	1.990	2.373	2.638	3.415
82	1.292	1.664	1.989	2.373	2.637	3.413
83	1.292	1.663	1.989	2.372	2.636	3.412
84	1.292	1.663	1.989	2.372	2.636	3.410
85	1.292	1.663	1.988	2.371	2.635	3.409
86	1.291	1.663	1.988	2.370	2.634	3.407
87	1.291	1.663	1.988	2.370	2.634	3.406
88	1.291	1.662	1.987	2.369	2.633	3.405
89	1.291	1.662	1.987	2.369	2.632	3.403
90	1.291	1.662	1.987	2.368	2.632	3.402
91	1.291	1.662	1.986	2.368	2.631	3.401
92	1.291	1.662	1.986	2.368	2.630	3.399
93	1.291	1.661	1.986	2.367	2.630	3.398
94	1.291	1.661	1.986	2.367	2.629	3.397
95	1.291	1.661	1.985	2.366	2.629	3.396
96	1.290	1.661	1.985	2.366	2.628	3.395
97	1.290	1.661	1.985	2.365	2.627	3.394
98	1.290	1.661	1.984	2.365	2.627	3.393
99	1.290	1.660	1.984	2.365	2.626	3.392
100	1.290	1.660	1.984	2.364	2.626	3.390
120	1.289	1.658	1.980	2.358	2.617	3.373
140	1.288	1.656	1.977	2.353	2.611	3.361
160	1.287	1.654	1.975	2.350	2.607	3.352
180	1.286	1.653	1.973	2.347	2.603	3.345
200	1.286	1.653	1.972	2.345	2.601	3.340
∞	1.282	1.645	1.960	2.326	2.576	3.291

B.6A Critical Values of the *F* Distribution ($\alpha = .05$)

Degrees of Freedom for the Denominator	Degrees of Freedom for the Numerator															
	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40
1	161	200	216	225	230	234	237	239	241	242	244	246	248	249	250	251
2	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4	19.4	19.5	19.5	19.5
3	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39