UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF STATISTICS

MODULE:	Analytical Techniques B (ATE01B1 / ATE1B10)
CAMPUS:	APK / DFC / SWC
ASSESSMENT:	Final Summative Assessment, Semester 2, 2018
DATE:	24 November 2018
DURATION:	2 hours (08:30 – 10:30)
MARKS:	40 = 100%
ASSESSOR(S):	Miss E Smit, Mrs W Rabotho
INTERNAL MODERATOR:	Mrs A Hildebrand

INSTRUCTIONS:

- This paper consists of 21 pages (excluding this one)
- This paper consists of 40 multiple choice questions
- Select only one option for each question
- Answer all questions on Blackboard
- The use of a non-programmable calculator is allowed
- Round off all intermediate and final answers to 3 decimal places, unless otherwise stated
- Formulae are provided on pages 14 – 15
- Statistical tables are provided on page 16 – 18
- Blank pages for rough work are provided on pages 19 – 21

LEARNER INFORMATION:

INITIALS AND SURNAME:	
STUDENT NUMBER:	

QUESTION 1

The drying time of a certain type of paint is assumed to follow a normal distribution with a standard deviation of 20 minutes. How many samples of paint must be tested to be 95% confident that the estimate of the mean drying time is within 4 minutes of the true mean? Round your answer to the nearest whole number.

a) 10
b) 41
c) 49
d) 96

QUESTION 2

With reference to Question 1:

Which sampling method would be most appropriate to select a sample of paint cans to test?

a) Simple random sampling
b) Stratified random sampling
c) Judgement sampling
d) Quota sampling
e) Snowball sampling

QUESTION 3

A textile manufacturer has 20 factories located throughout South Africa, and each factory has 40 sewing machines (numbered 1 to 40). The manufacturer wants to estimate the proportion of sewing machines that are still in good working condition. To do this, they need to select a sample of machines and assess the condition of each. Since the factories are widely scattered, a cluster sampling approach is used to select a sample of factories (clusters). Within each cluster, 25% of the machines at the factory are selected through systematic sampling. The total sample size is $n = 50$. Which one of the following statements is incorrect?

a) A total of 10 machines are selected from each factory
b) A total of 5 factories are selected
c) At each factory, every 4^{th} machine is selected
d) If machine number 4 is the first machine selected at a factory, machine number 40 will not be included in the sample

QUESTION 4

A manufacturer of boxes of matches claims that there is an average of 48 matches per box produced at this factory. A customer believes that this claim is not true. He randomly selected a sample of 16 boxes and counted the number of matches. It is generally assumed that the number of matches per box is normally distributed with a standard deviation of 2. Which one of the following statements is true?

a) The sampling distribution of the mean of this sample is not normally distributed because the sample is less than 30
b) The standard error of the mean of this sample is equal to 0.5
c) A larger sample size would yield a larger standard error of the mean
d) The sample mean is equal to 48

QUESTION 5

With reference to Question 4:

The correct null and alternative hypotheses to represent the customer's belief is given as:

a) $H_0 : \overline{x} = 48$ vs. $H_1 : \overline{x} < 48$
b) $H_0 : \overline{x} = 48$ vs. $H_1 : \overline{x} \neq 48$
c) $H_0 : \mu = 48$ vs. $H_1 : \mu > 48$
d) $H_0 : \mu = 48$ vs. $H_1 : \mu \neq 48$

QUESTION 6

With reference to Question 4:

The 95% confidence interval for the average number of matches from the sample of 16 boxes was found to be (45.42, 47.38). Consider the following statements and choose the correct alternative from a) to e) below:

I. The point estimate is equal to 46.4
II. We are 95% confident that the average number of matches per box produced by the manufacturer is between 45.42 and 47.38
III. A 90% confidence interval will include the value of $\mu = 48$, claimed by the manufacturer to be the population average

a) Only statements I and II are true
b) Only statements II and III are true
c) Only statements I and III are true
d) All statements I, II and III are true
e) All statements I, II and III are false

QUESTION 7

A construction company supplies its employees with gloves that have an average life of 90 days. A new type of glove recommended by a glove manufacturer is believed to last longer, even though it is more expensive. The construction company decided to test the new gloves through a pilot program. A sample of 20 employees used the new gloves for a period of time and the mean lifetime was found to be 95 days with a standard deviation 10 days. It is assumed that lifetime of the gloves is approximately normally distributed. The 95% confidence interval of the mean lifetime of the new gloves is give as:

a) $90 \pm 1.729 \frac{10}{\sqrt{20}}$

b) $90 \pm 1.960 \frac{10}{\sqrt{20}}$

c) $90 \pm 2.093 \frac{10}{\sqrt{20}}$

d) $95 \pm 1.960 \frac{10}{\sqrt{20}}$

e) $95 \pm 2.093 \frac{10}{\sqrt{20}}$

QUESTION 8

With reference to Question 7:

In testing the hypothesis that the new gloves lasts longer than 90 days, on average, the following information is given:

Calculated test statistic value	Critical values for given levels of significance		
	1%	5%	10%
2.236	2.438	1.690	1.306

Based on the information in the table above, which one of the following intervals shows the correct size of the *p*-value for this test?

a) $0 < p\text{-value} < 0.01$

b) $0.01 < p\text{-value} < 0.05$

c) $0.05 < p\text{-value} < 0.1$

d) $0.1 < p\text{-value} < 1$

QUESTION 9

Historically, the main product of a certain company had a 20% market share. To increase sales, the company conducted an extensive advertising campaign. At the end of the campaign a sample of 130 potential customers were interviewed and 25% indicated that they would purchase the product in future. The margin of error of a 99% confidence interval of the proportion of potential customers that will buy their main product after the advertising campaign is equal to:

a) 0.035
b) 0.038
c) 0.090
d) 0.098
e) 2.576

QUESTION 10

A company has set up a criterion for accepting a shipment of a product. It will reject a shipment if more than 5% of the shipment is defective. A shipment consisting of 200 items has just arrived. The quality control manager randomly selects a sample of 50 items and finds 2 to be defective. In testing if there is evidence that the company should reject the shipment at the 5% level of significance, which one of the following statements is false?

a) H_0: $\pi = 0.05$ vs. H_1: $\pi > 0.05$
b) Critical value = +1.645
c) Test statistic = −0.649
d) There is not enough evidence to reject the shipment

QUESTION 11

A random variable X is assumed to follow a normal distribution. A random sample of 25 observations yielded the following statistics:

$\bar{x} = 1550$ and $s = 125$

A hypothesis test is performed at the 10% level of significance to see if these statistics provide any evidence to indicate that μ is not equal to 1500. The calculated test statistic is equal to 2.53. Which one of the following statements is incorrect?

a) This is a one-sample *t*-test
b) The critical value is equal to ±1.711
c) The hypothesis that $\mu \neq 1500$ is rejected
d) Since X is normally distributed it follows that the sample mean is normally distributed

QUESTION 12

A pizza parlour noted that it takes on average 30 minutes to deliver pizzas if they make use of a fast food delivery service. The new manager at the pizza parlour claims that they can improve their delivery times if they employ delivery drivers in-house. This is an example of a:

a) One-tailed test to the left
b) One-tailed test to the right
c) Two-tailed test

QUESTION 13

With reference to Question 12:

The pizza parlour employed an experienced delivery driver and recorded his delivery times. The mean and standard deviation of a random sample of 25 deliveries by this driver was 28.9 minutes and 5 minutes respectively. It is assumed that delivery times are normally distributed with a standard deviation of 4.5 minutes. The calculated test statistic is equal to:

a) −1.222
b) +1.222
c) −1.100
d) +1.100

QUESTION 14

With reference to Question 12:

In testing the hypothesis at the 1% level of significance, which one of the following statements is true?

a) There is insufficient evidence that the fast food delivery service is better than the new driver
b) There is insufficient evidence that the new driver is better than the fast food delivery service
c) There is insufficient evidence the that new driver is the same as the fast food delivery service
d) There is sufficient evidence that the new driver is worse than the fast food delivery service

QUESTION 15

A software company wants to evaluate how various features of laptops, such as processor speed, memory and size of the hard drive, influence its purchase price. Which technique can be uses to determine this?

a) Chi-squared goodness-of-fit
b) Chi-squared test of independence
c) Multiple linear regression
d) One-way ANOVA

QUESTION 16

A flooring company, specialising in laminate flooring, operates in three major municipalities in Gauteng: City of Johannesburg, City of Tshwane and Ekurhuleni. The company suspects that consumers' choice of using laminate flooring vs. other floor coverings is dependent on geographic region. To test this, they conducted a survey of 400 recently built homes across the three regions and recorded the new home owners' choice of floor covering. The data are summarised as follows:

Observed frequencies		Geographic region (Municipality)			
		City of Johannesburg	City of Tshwane	Ekurhuleni	Total
Type of floor covering	Laminate	?	62	36	178
	Other	126	78	18	222
	Total	206	140	54	400

If the choice of floor covering type is independent of geographic region, how many new home owners are expected to choose laminate flooring in the City of Johannesburg?

a) 80.00
b) 66.67
c) 91.67
d) 103.00

QUESTION 17

With reference to Question 16:

In testing the appropriate hypothesis at the 5% level of significance, the critical value is equal to:

a) 2.920
b) 5.991
c) 7.378
d) 12.592

QUESTION 18

With reference to Question 16:

An analysis of the data above yielded a p-value of 0.0006. At the 1% level of significance:

a) H_0 is not rejected, therefore the choice of floor covering is dependent on geographic region
b) H_0 is not rejected, therefore the choice of floor covering is independent on geographic region
c) H_0 is rejected, therefore the choice of floor covering is independent of geographic region
d) H_0 is rejected, therefore the choice of floor covering is dependent on geographic region

QUESTION 19

SANRAL wants to know if some lanes on a particular stretch of the N17 highway between Boksburg and Benoni are preferred over others. At a certain point on the N17, a total of 110 cars were observed in the slow lane, 100 in the middle lane, and 90 in the fast lane. Consider the following statements and choose the correct alternative from a) to d) below:

I. H_0: there is an equal number of vehicles in each lane

II. The calculated test statistic is equal to 3

a) Only statement I is correct

b) Only statement II is correct

c) Both statements I and II are correct

d) None of statements are correct

QUESTION 20

The records of an investment banking firm show that, historically, 60% of its clients were primarily interested in the stock market, 36% in the bonds market, and 4% in the futures market. A recent study of 200 clients showed that 132 were primarily interested in stocks, 52 in bonds and 16 in futures. Selected results of a Chi-squared goodness-of-fit test are given as follows:

	Stocks	Bonds	Futures	Total
Observed frequency	130	60	10	200
Expected frequency	**A**	?	8	?
$\frac{(f_o - f_e)^2}{f_e}$	?	?	**B**	3.33

Which one of the following statements is incorrect?

a) The value of A in the table is equal to 120

b) The value of B in the table is equal to 0.25

c) The critical value for $\alpha = 0.1$ is equal to 4.605

d) There is insufficient evidence at the 10% level of significance to show that there is a shift in the primary interest of clients

QUESTION 21

A researcher suspects that background noise affects a learner's ability to absorb what they are learning. She randomly divides students into three equal groups. Students in Group 1 study with background sound at a constant volume, students in Group 2 study with background sound that changes volume periodically, and students in Group 3 study with no background sound at all. The students all study a passage of text for 30 minutes. After studying, all students write a 10-point multiple choice test on the material. Assume that test scores are approximately normally distributed. The analysis of variance results are as follows:

	SS	*df*	*MS*	*F*
Between	30.08	?	15.04	3.60
Within	?	21	4.18	
Total	117.96	?		

The sum of squares within groups is equal to:

a) 100.04

b) 97.92

c) 87.88

d) 46.92

QUESTION 22

With reference to Question 21:

How many students were included in this study in total?

a) 16

b) 18

c) 20

d) 22

e) 24

QUESTION 23

With reference to Question 21:

For $\alpha = 0.05$ the critical value is equal to:

a) 3.47

b) 3.42

c) 3.07

d) 3.03

QUESTION 24

With reference to Question 21:

When $\alpha = 0.025$, the critical value is 4.42. Which one of the following statements is correct?

a) Reject the null hypothesis because $4.42 > 3.60$

b) Do not reject the null hypothesis because $3.60 < 4.42$

c) Accept the alternative hypothesis because $3.60 < 4.42$

d) Do not reject the alternative hypothesis because $4.42 > 3.60$

QUESTION 25

A telesales company wants to predict the productivity of its employees. The company considers its employees experience in months, age in years and productivity (measured as the number of sales made). The manager of the company randomly selected employees and after a week their information were analysed. The following output shows the results of a multiple linear regression of the number of sales on age and experience.

Regression Statistics	
Multiple R	0.943
R Square	0.889
Adjusted R Square	0.876
Standard Error	2.974
Observations	20

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	17.228	10.430	1.625	0.117
Experience	1.468	0.192	7.664	0.000
Age	−0.898	0.525	−1.710	0.106

What proportion of variation in productivity is explained by variation in the age and experience?

a) 0.117

b) 0.889

c) 0.876

d) 0.943

QUESTION 26

With reference to Question 25:

Which of the two predictor variables had a significant impact on the productivity?

a) Experience only
b) Age only
c) Both experience and age
d) Neither experience nor age

QUESTION 27

With reference to Question 25:

If age is increased by one year while experience in months is held constant, productivity is predicted to:

a) Increase by one unit
b) Decrease by one unit
c) Increase by 0.898
d) Decrease by 0.898

QUESTION 28

With reference to Question 25:

Find the predicted productivity of an employee who is 27 years old and has 20 months experience.

a) 22.342
b) 38.904
c) 53.606
d) 70.834

QUESTION 29

Find the derivative of $y = 4x^{\frac{3}{2}} + x^{\frac{1}{2}}$

a) $\frac{6}{\sqrt{x}}$
b) $\frac{8}{\sqrt{x}}$
c) $6\sqrt{x} + \frac{1}{\sqrt{x}}$
d) $\frac{6}{\sqrt{x} + x}$

QUESTION 30

If $y=\frac{2x-1}{4x+3}$, then $\frac{dy}{dx}$ is equal to:

a) $\frac{10x+3}{(4x+3)^2}$

b) $\frac{8x+6}{(4x+3)^2}$

c) $\frac{2x-1}{(4x+3)^2}$

d) $\frac{12}{(4x+3)^2}$

e) $\frac{10}{(4x+3)^2}$

QUESTION 31

Find the second order derivative of $y=2e^{-4x}$

a) $y''=-32e^{-4x}$

b) $y''=+32e^{-4x}$

c) $y''=-8e^{-4x}$

d) $y''=+8e^{-4x}$

e) $y''=-16e^{-4x}$

QUESTION 32

If $f(x)=\left(-3\sqrt{x}\right)\left(4\sqrt{x}-2\right)$, then $f'(x)$ is equal to:

a) $-12+\frac{3}{\sqrt{x}}$

b) $-12+3\sqrt{x}$

c) $-12\sqrt{x}+\frac{3}{\sqrt{x}}$

d) $-9\sqrt{x}$

QUESTION 33

If $y = \ln\left(\frac{x}{x+2}\right)$, then the derivative of y with respect to x is equal to:

a) $\frac{1}{x+1}$

b) $\frac{x+2}{x}$

c) $\frac{2}{x(x+2)}$

d) $\frac{2x+2}{x(x+2)}$

QUESTION 34

For the function $y = x^2 + 6x + 2$, which one of the following statements is correct?

a) There is a maximum turning point at -3

b) There is a maximum turning point at $+2$

c) There is a minimum turning point at $+3$

d) There is a minimum turning point at -3

QUESTION 35

The profit function (in R'000) from the sale of x thousand vehicle tyres is approximated by the function $P(x) = -x^3 + 9x^2 + 120x - 400$, where $x \geq 5$. The number of tyres that will maximize the profit is equal to:

a) 4000

b) 8000

c) 9500

d) 10000

QUESTION 36

Find the (x, y)-coordinate of the inflection point of the function $f(x) = x^3 + 3x^2 + 5$.

a) $(1, 8)$

b) $(-1, 7)$

c) $(-1, 9)$

d) $(-2, 9)$

QUESTION 37

A jeweller makes rings and watches. Each ring is sold for a profit of R20 and each watch for a profit of R10. The jeweller can afford to spend up to 40 hours per week working. It takes the jeweller six hours to make a ring and three hours to make a watch. The jeweller makes three times as many watches as rings. Let r = number of rings and w = number of watches. The constraint for total work time is equal to:

a) $6r+3w \leq 30$

b) $3r+6w \leq 40$

c) $6r+3w \leq 40$

d) $20r+10w \leq 40$

QUESTION 38

With reference to Question 37:

The objective function (z) of profit is:

a) $z=r+3w$

b) $z=3r+w$

c) $z=10r+20w$

d) $z=20r+10w$

QUESTION 39

Consider an objective function $z=3x+4y$, subject to constraints. The feasible region based on the constraints are bounded by vertices A, B, C and D. The (x, y)-coordinates of the four vertices are as follows:

A = (0, 11) B = (8, 3) C = (5, 0) D = (0, 0)

The optimal solution of the objective function is found at the point/vertex:

a) A

b) B

c) C

d) D

QUESTION 40

Consider the following statement: "*The second order derivative test can be used to decide whether a given function's critical values produce maximum and/or minimum values*".

a) This statement is true

b) The statement is false

FORMULAE SHEET AND STATISTICAL TABLES

SAMPLING DISTRIBUTION OF THE MEAN

Expected value	$E(\bar{X}) = \mu$
Variance	$\text{Var}(\bar{X}) = \frac{\sigma^2}{n}$
Standard error	$\frac{\sigma}{\sqrt{n}}$

INFERENCE

Problem statement	Sample size calculation	Point estimate	Confidence interval	Null hypothesis	Test statistic
Single mean, σ or σ^2 known	$n = \frac{z^2\sigma^2}{e^2}$	$\bar{x} = \frac{1}{n}\sum x$	$\bar{x} \pm z\frac{\sigma}{\sqrt{n}}$	$H_0 : \mu = \mu_0$	$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1)$
Single mean, σ or σ^2 unknown			$\bar{x} \pm t\frac{s}{\sqrt{n}}$	$H_0 : \mu = \mu_0$	$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} \sim t_{n-1}$
Single proportion	$n = \frac{z^2 p(1-p)}{e^2}$	$p = \frac{x}{n}$	$p \pm z\sqrt{\frac{p(1-p)}{n}}$	$H_0 : \pi = \pi_0$	$z = \frac{p - \pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}} \sim N(0,1)$

CHI-SQUARED

Degrees of freedom	Test of independence	$df = (r-1)\times(c-1)$
	Goodness-of-fit test	$df = (k-1)$
Expected value		$e_{ij} = \frac{(\text{row } i \text{ total})\times(\text{column } j \text{ total})}{n}$
Test statistic		$\chi^2_{\text{test}} = \sum\frac{(f_o - f_e)^2}{f_e}$

MULTIPLE LINEAR REGRESSION

Least squares regression line	$\hat{y} = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + \ldots + b_px_p$

ANALYSIS OF VARIANCE

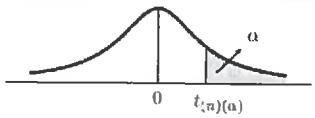
Total sum of squares	$SS_{\text{Total}} = SST + SSE$
Degrees of freedom 1 (Treatment)	$df_1 = k - 1$
Degrees of freedom 2 (Error)	$df_2 = N - k$
Mean square (Treatment)	$MST = \dfrac{SST}{df_1}$
Mean square (Error)	$MSE = \dfrac{SSE}{df_2}$
F-statistic	$F_{\text{stat}} = \dfrac{MST}{MSE}$

DIFFERENTIATION

Derivative of a constant	If $y = k$ then $\dfrac{dy}{dx} = 0$
Power rule	If $y = x^n$ then $\dfrac{dy}{dx} = nx^{n-1}$ If $y = \left[f(x)\right]^n$ then $\dfrac{dy}{dx} = n\left[f(x)\right]^{n-1} f'(x)$
Derivative of a sum	If $y = u + v$ then $\dfrac{dy}{dx} = \dfrac{du}{dx} + \dfrac{dv}{dx}$
Product rule	If $y = u \times v$ then $\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
Quotient rule	If $y = \dfrac{u}{v}$ then $\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$
Derivative of the natural exponential function	If $y = e^x$ then $\dfrac{dy}{dx} = e^x$ If $y = e^{f(x)}$ then $\dfrac{dy}{dx} = e^{f(x)} f'(x)$
Derivative of the natural logarithmic function	If $y = \ln x$ then $\dfrac{dy}{dx} = \dfrac{1}{x}$ If $y = \ln f(x)$ then $\dfrac{dy}{dx} = \dfrac{1}{f(x)} f'(x)$
Chain rule	If $y = f\left(g(x)\right)$ and $u = g(x)$ then $\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$

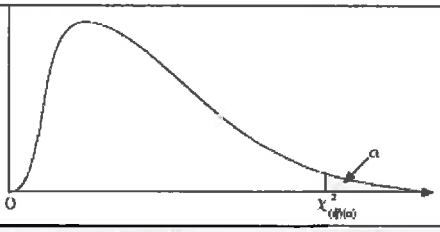
The Student's t distribution

This table gives the value of $t_{(n)(\alpha)}$ where n is the degrees of freedom i.e. $= P(t \geq t_{(n)(\alpha)})$



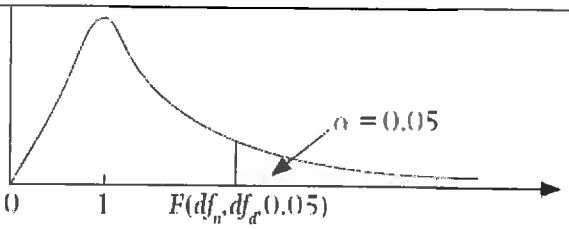
df \ α	0.10	0.050	0.025	0.01	0.005	0.0025
1	3.078	6.314	12.706	31.821	63.657	127.322
2	1.886	2.920	4.303	6.965	9.925	14.089
3	1.638	2.353	3.182	4.541	5.841	7.453
4	1.533	2.132	2.776	3.747	4.604	5.598
5	1.476	2.015	2.571	3.365	4.032	4.773
6	1.440	1.943	2.447	3.143	3.707	4.317
7	1.415	1.895	2.365	2.998	3.499	4.029
8	1.397	1.860	2.306	2.896	3.355	3.833
9	1.383	1.833	2.262	2.821	3.250	3.690
10	1.372	1.812	2.228	2.764	3.169	3.581
11	1.363	1.796	2.201	2.718	3.106	3.497
12	1.356	1.782	2.179	2.681	3.055	3.428
13	1.350	1.771	2.160	2.650	3.012	3.372
14	1.345	1.761	2.145	2.624	2.977	3.326
15	1.341	1.753	2.131	2.602	2.947	3.286
16	1.337	1.746	2.120	2.583	2.921	3.252
17	1.333	1.740	2.110	2.567	2.898	3.222
18	1.330	1.734	2.101	2.552	2.878	3.197
19	1.328	1.729	2.093	2.539	2.861	3.174
20	1.325	1.725	2.086	2.528	2.845	3.153
21	1.323	1.721	2.080	2.518	2.831	3.135
22	1.321	1.717	2.074	2.508	2.819	3.119
23	1.319	1.714	2.069	2.500	2.807	3.104
24	1.318	1.711	2.064	2.492	2.797	3.091
25	1.316	1.708	2.060	2.485	2.787	3.078
26	1.315	1.706	2.056	2.479	2.779	3.067
27	1.314	1.703	2.052	2.473	2.771	3.057
28	1.313	1.701	2.048	2.467	2.763	3.047
29	1.311	1.699	2.045	2.462	2.756	3.038
30	1.310	1.697	2.042	2.457	2.750	3.030
31	1.309	1.696	2.040	2.453	2.744	3.022
32	1.309	1.694	2.037	2.449	2.738	3.015
33	1.308	1.692	2.035	2.445	2.733	3.008
34	1.307	1.691	2.032	2.441	2.728	3.002
35	1.306	1.690	2.030	2.438	2.724	2.996
36	1.306	1.688	2.028	2.434	2.719	2.990
37	1.305	1.687	2.026	2.431	2.715	2.985
38	1.304	1.686	2.024	2.429	2.712	2.980
39	1.304	1.685	2.023	2.426	2.708	2.976
40	1.303	1.684	2.021	2.423	2.704	2.971
45	1.301	1.679	2.014	2.412	2.690	2.952
50	1.299	1.676	2.009	2.403	2.678	2.937
60	1.296	1.671	2.000	2.390	2.660	2.915
70	1.294	1.667	1.994	2.381	2.648	2.899
80	1.292	1.664	1.990	2.374	2.639	2.887
90	1.291	1.662	1.987	2.368	2.632	2.878
100	1.290	1.660	1.984	2.364	2.626	2.871
110	1.289	1.659	1.982	2.361	2.621	2.865
120	1.289	1.658	1.980	2.358	2.617	2.860
140	1.288	1.656	1.977	2.353	2.611	2.852
160	1.287	1.654	1.975	2.350	2.607	2.846
180	1.286	1.653	1.973	2.347	2.603	2.842
200	1.286	1.653	1.972	2.345	2.601	2.839
∞	1.282	1.645	1.960	2.327	2.576	2.807

The Chi-squared distribution
This table gives the value of $\chi^2_{(df)(\alpha)}$ where df is the degrees of freedom i.e. $\alpha = P(\chi^2 \geq \chi^2_{(df)(\alpha)})$



df/α	0.100	0.050	0.025	0.010	0.005	0.0025
1	2.707	3.843	5.026	6.637	7.881	9.142
2	4.605	5.991	7.378	9.210	10.597	11.983
3	6.251	7.815	9.348	11.345	12.838	14.320
4	7.779	9.488	11.143	13.277	14.860	16.424
5	9.236	11.070	12.833	15.086	16.750	18.386
6	10.645	12.592	14.449	16.812	18.548	20.249
7	12.017	14.067	16.013	18.475	20.278	22.040
8	13.362	15.507	17.535	20.090	21.955	23.774
9	14.684	16.919	19.023	21.666	23.589	25.462
10	15.987	18.307	20.483	23.209	25.188	27.112
11	17.275	19.675	21.920	24.725	26.757	28.729
12	18.549	21.026	23.337	26.217	28.300	30.318
13	19.812	22.362	24.736	27.688	29.819	31.883
14	21.064	23.685	26.119	29.141	31.319	33.426
15	22.307	24.996	27.488	30.578	32.801	34.950
16	23.542	26.296	28.845	32.000	34.267	36.456
17	24.769	27.587	30.191	33.409	35.718	37.946
18	25.989	28.869	31.526	34.805	37.156	39.422
19	27.204	30.144	32.852	36.191	38.582	40.885
20	28.412	31.410	34.170	37.566	39.997	42.336
21	29.615	32.671	35.479	38.932	41.401	43.775
22	30.813	33.924	36.781	40.289	42.796	45.204
23	32.007	35.172	38.076	41.638	44.181	46.623
24	33.196	36.415	39.364	42.980	45.559	48.034
25	34.382	37.652	40.646	44.314	46.928	49.435
26	35.563	38.885	41.923	45.642	48.290	50.829
27	36.741	40.113	43.195	46.963	49.645	52.215
28	37.916	41.337	44.461	48.278	50.993	53.594
29	39.087	42.557	45.722	49.588	52.336	54.967
30	40.256	43.773	46.979	50.892	53.672	56.332
31	41.422	44.985	48.232	52.191	55.003	57.692
32	42.585	46.194	49.480	53.486	56.328	59.046
33	43.745	47.400	50.725	54.776	57.648	60.395
34	44.903	48.602	51.966	56.061	58.964	61.738
35	46.059	49.802	53.203	57.342	60.275	63.076
36	47.212	50.998	54.437	58.619	61.581	64.410
37	48.363	52.192	55.668	59.893	62.883	65.739
38	49.513	53.384	56.896	61.162	64.181	67.063
39	50.660	54.572	58.120	62.428	65.476	68.383
40	51.805	55.758	59.342	63.691	66.766	69.699

The F distribution ($\alpha = 0.05$)
Critical value for F for which the area under the curve to the right is equal to 0.05



Degrees of freedom for DENOMINATOR	Degrees of freedom for NUMERATOR									
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91
∞	3.84	3.00	2.61	2.37	2.21	2.10	2.01	1.94	1.88	1.83

ROUGH WORK

ROUGH WORK

ROUGH WORK