UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF PURE AND APPLIED MATHEMATICS

MODULE: ASME1A1

COURSE: CALCULUS OF ONE VARIABLE FUNCTIONS FOR ENGINEERS (ALTERNATIVE SEMESTER)

CAMPUS: APK

EXAM: JANUARY 2019

DATE: TBA

TIME: TBA

ASSESSOR: MR W VAN REENEN

INTERNAL MODERATOR: DR A CRAIG

DURATION: 2 HOURS

MARKS: 70

SURNAME AND INITIALS

STUDENT NUMBER

CONTACT NUMBER

NUMBER OF PAGES: 1+13 PAGES (including front page)

INSTRUCTIONS: ANSWER ALL THE QUESTIONS IN PEN

NO CALCULATORS ALLOWED.

If you require extra space, continue on the adjacent blank page next to it and indicate this clearly.

Question 1 [10 marks]

For questions 1.1 – 1.10, choose **one** correct answer, and make a cross (X) in the correct block.

Question	a	b	c	d	e
1.1					
1.2					
1.3					
1.4					
1.5					
1.6					
1.7					
1.8					
1.9					
1.10					

1.1 Solving $|x-1| = |x+1|$ yields: [1]

(a) $x = \pm 2$

(b) $x = \pm 1$

(c) $x = 0$

(d) No solutions.

(e) None of the above

1.2 The correct expansion of $\sum_{i=3}^{5} (-1)^i \frac{3^i}{2^i}$ is: [1]

(a) $-\frac{27}{4} - \frac{81}{8} - \frac{243}{16}$

(b) $-\frac{27}{8} + \frac{81}{16} - \frac{243}{32}$

(c) $-\frac{9}{8} + \frac{27}{16} - \frac{81}{32}$

(d) $-\frac{27}{8} - \frac{81}{16} - \frac{243}{32}$

(e) None of the above

1.3 $1 + \tan^2 x$ equals: [1]

(a) $\csc x$

(b) $\sec x$

(c) $1 - \sin^2 x$

(d) $\tan^2 x$

(e) None of the above

1.4 If $f(x) = x^4 - 1$, $g(x) = \sqrt[4]{x^2 - 1}$ and $h(x) = \sqrt{x + 2}$, then $(f \circ g \circ h)(x)$ equals: [1]

(a) x^2

(b) x

(c) $2x$

(d) $x - 1$

(e) None of the above

1.5 Solving $|2x - 5| < 3$ yields: [1]

(a) $x \leq -1$ or $x \geq 4$

(b) $-4 < x < -1$

(c) $x < 4$ or $x > -1$

(d) $1 < x < 4$

(e) None of the above

1.6 The derivative of $y = 10^x$ is: [1]

(a) $x10^{x-1}$

(b) $10^x \ln x$

(c) $10^x \ln 10$

(d) $\ln 10^x . \frac{1}{10}$

(e) None of the above

1.7 If f is differentiable, then $\frac{d}{dx}(f(\sqrt{x})) =$ [1]

(a) $\frac{f'(x)}{2\sqrt{x}}$

(b) $f'(x)\sqrt{x}$

(c) $\frac{f'(\sqrt{x})}{2\sqrt{x}}$

(d) $\frac{f'(x)}{\frac{1}{2}\sqrt{x}}$

(e) None of the above

1.8 The negation of the conditional proposition $p \rightarrow q$ is: [1]

(a) $\neg p \wedge \neg q$

(b) $p \vee \neg q$

(c) $\neg p \vee \neg q$

(d) $p \wedge \neg q$

(e) None of the above

1.9 Which of the following is a counter example or the statement: "If n is a non negative integer, then $2^n + 1$ is a prime number". [1]

(a) 1

(b) 2

(c) 3

(d) 4

(e) None of the above

1.10 If $\int_0^9 f(x)dx = 12$ and $\int_0^9 g(x)dx = 16$, then $\int_0^9 (2f(x) + 3g(x))\,dx =$ [1]

(a) 72

(b) 68

(c) 252

(d) 28

(e) None of the above

Question 2 [8 marks]

(a) Rewrite the first order statement in natural language: $(\forall x \in \mathbb{R})(x > 2 \rightarrow x^2 > 4)$ [1]

(b) Write the negation of the sentence given in 2(a) using natural language. [2]

(c) Determine whether the following statement is true or false: [2]
For any positive integers m and n, $mn = m + n$

(d) Use proof by contraposition to show that if $\sin(\frac{k\pi}{2}) = -1$, then $k = 7$ [3]

Question 3 [3 marks]

Solve for x if: $\dfrac{2x^2}{x+5} \leq \dfrac{x}{x+5} + \dfrac{3}{x+5}$. [3]

Question 4 [3 marks]

If $\tan\theta = -\sqrt{3}$, $\pi < \theta < 2\pi$, find the five other trigonometric ratios. [3]

Question 5 [7 marks]

(a) If f is the function defined below, determine whether f is even, odd, or neither: [2]
$f(x) = 2 - 2\sin x$

(b) Determine the inverse of $f(x) = 2\ln(2 - e^x)$. [3]

(c) Sketch the graph of $y = \cot\theta$, for $0 < \theta < 2\pi$. Indicate any intercepts and asymptotes. [2]

Question 6 [4 marks]

Let f be the function defined by $f(x) = \ln(x+1) + 1$,

(a) State the domain. [1]

(b) State the range. [1]

(c) Sketch the graph of f by making use of translations. Draw a separate sketch for each transformation. Mark any asymptotes or intercepts with the axes. [2]

Question 7 [4 marks]

Determine the following limits:

(a) $\lim_{x \to 2} \dfrac{x^3 - 8}{x - 2}$ [2]

(b) $\lim_{x \to \infty} \dfrac{\sqrt{9x^8 - x + 1}}{3x^4 + 5x}$ [2]

Question 10 [5 marks]

(a) State the Fundamental Theorem of Calculus (Part 1 and 2). [2]

(b) Use part 1 of the theorem to find $g'(x)$ given that $g(x) = \int_1^{3\sqrt{x}} \frac{3t^2}{1+t}\,dt$. [3]

Question 11 [6 marks]

Evaluate the following integrals:

(a) $\int_0^2 \frac{x}{x+1}\,dx$ [3]

(b) $\int x^5(x^3+2)^2 dx$ [3]

Question 12 [4 marks]

Prove the following statement: "If f is differentiable at a, then f is continuous at a". [4]

EXTRA Work sheet