



## FACULTY OF SCIENCE

### DEPARTMENT OF PURE AND APPLIED MATHEMATICS

**MODULE:** ASME1A1

**COURSE:** CALCULUS OF ONE VARIABLE FUNCTIONS FOR ENGINEERS  
(ALTERNATIVE SEMESTER)

**CAMPUS:** APK

**EXAM:** NOVEMBER 2018

**DATE:** TBA

**TIME:** TBA

**ASSESSOR:**

**MR W VAN REENEN**

**INTERNAL MODERATOR:**

**DR A CRAIG**

**DURATION:** 2 HOURS

**MARKS:** 70

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**SURNAME AND INITIALS**

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**STUDENT NUMBER**

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**CONTACT NUMBER**

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**NUMBER OF PAGES:** 1+12 PAGES (including front page)

**INSTRUCTIONS:** ANSWER ALL THE QUESTIONS IN PEN  
NO CALCULATORS ALLOWED.

If you require extra space, continue on the adjacent blank page next to it  
and indicate this clearly.

ASME1A1 NOVEMBER EXAM

Question 1 [9 marks]

For questions 1.1 - 1.9, choose **one** correct answer, and make a cross (X) in the correct block.

Question	a	b	c	d	e
1.1					
1.2					
1.3					
1.4					
1.5					
1.6					
1.7					
1.8					
1.9					

1.1 Which of the following is not an exact ratio for  $\frac{2\pi}{3}$ ? [1]

(a)  $\sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2}$

(b)  $\cot \frac{2\pi}{3} = \frac{1}{\sqrt{3}}$

(c)  $\csc \frac{2\pi}{3} = \frac{2}{\sqrt{3}}$

(d)  $\sec \frac{2\pi}{3} = -2$

(e) None of the above

1.2 Which of the following is not an identity of  $\cos 2x$ ? [1]

(a)  $2\cos^2 x + 1$

(b)  $\cos^2 x - \sin^2 x$

(c)  $1 - 2\sin^2 x$

(d)  $-\ln e + 2\cos^2 x$

(e) None of the above

1.3 The correct expansion of  $\sum_{i=1}^3 (-1)^{i+1} \frac{3^i}{i+1}$  is: [1]

(a)  $-\frac{3}{2} + \frac{3^2}{3} - \frac{3^3}{4}$

(b)  $\frac{3}{2} + \frac{3^2}{3} + \frac{3^3}{4}$

(c)  $\frac{3}{2} - \frac{3^3}{4} + \frac{3^4}{5}$

(d)  $-\frac{3}{2} - \frac{3^2}{3} - \frac{3^3}{4}$

(e) None of the above

1.4 If  $f(x) = x^3 - 1$ ,  $g(x) = \sqrt[3]{x^2 - 1}$  and  $h(x) = \sqrt{x + 2}$ , then  $(f \circ g \circ h)(x)$  equals: [1]

(a)  $\sqrt{x}$

(b)  $x$

(c)  $\sqrt{x + 1}$

(d)  $x + 1$

(e) None of the above

1.5 Solving  $|7x + 1| > 2$  yields: [1]

(a)  $x > -\frac{2}{7}$  or  $x < \frac{1}{7}$

(b)  $-\frac{2}{7} \leq x \leq \frac{1}{7}$

(c)  $x < -\frac{2}{7}$  or  $x > \frac{1}{7}$

(d)  $x \leq -\frac{2}{7}$  or  $x > \frac{1}{7}$

(e) None of the above

1.6 Which of the following is not a proposition? [1]

- (a) UJ is a university
- (b) Durban is the capital of South Africa
- (c) 8 minus 5 equals 10
- (d)  $x$  is an integer
- (e) None of the above

1.7 The contrapositive of  $\neg p \rightarrow q$  is: [1]

- (a)  $q \rightarrow p$
- (b)  $p \rightarrow q$
- (c)  $\neg q \rightarrow \neg p$
- (d)  $\neg p \rightarrow \neg q$
- (e) None of the above

1.8 The derivative of  $y = e^{\ln x} + \ln e^x$  is: [1]

- (a) 1
- (b)  $e$
- (c)  $\ln x$
- (d) 2
- (e) None of the above

1.9 Evaluating  $\int_{-1}^1 \frac{\tan x}{1+x^2+x^4} dx$  yields: [1]

- (a)  $\pi$
- (b)  $-1$
- (c) 0
- (d) 1
- (e) None of the above

Question 2 [8 marks]

Given the following case-defined function:

$$f(x) = \begin{cases} \ln x & \text{if } x > 1 \\ x^2 - 1 & \text{if } x < 1 \end{cases}$$

2.1 Graph the function

[2]

2.2 Determine:

2.2.1  $\lim_{x \rightarrow 1^-} f(x)$

[1]

2.2.2  $\lim_{x \rightarrow 1^+} f(x)$

[1]

2.2.3  $\lim_{x \rightarrow 1} f(x)$

[1]

2.2.4  $f(1)$

[1]

2.3 Is  $f$  differentiable at  $x = 1$ ? Explain using your results from 2.2.

[2]

Question 3 [4 marks]

Solve for  $x$  if:  $\frac{x}{x^2 + 3x - 4} < -\frac{3}{x^2 + 3x - 4}$

Question 4 [2 marks]

If  $g$  is the function defined below, determine whether  $g$  is odd, even or neither:

$$g(x) = 2 \sin x - 5 \cos x$$

Question 5 [2 marks]

Sketch the graph of  $y = \csc \theta$  for  $\theta \in (0, 2\pi)$ . Include all intercepts and asymptotes if any.

Question 6 [4 marks]

If  $j$  is the function defined by  $j(x) = 2e^{(x-1)} + 1$

5.1 Sketch the graph of  $f$  by making use of translations. Do a separate sketch for each transformation, showing clearly all intercepts. [2]

5.2 State the domain. [1]

5.3 State the range [1]

Question 7 [3 marks]

If  $m(x) = \frac{x+2}{x-2}$ , determine  $m'(x)$  by making use of first principles.

Question 8 [4 marks]

Determine:

(a)  $\lim_{x \rightarrow 1} \frac{x^4 - 1}{x - 1}$  [2]

(b)  $\lim_{x \rightarrow \infty} \frac{\sqrt{9x^4 - x}}{x^2 + 3}$  [2]

Question 9 [5 marks]

By completing the truth table below, Determine whether or not the following propositions are logically equivalent:

$p \rightarrow (q \rightarrow r)$  and  $(p \rightarrow q) \rightarrow r$

<b>p</b>	<b>q</b>	<b>r</b>				
T	T	T				
T	T	F				
T	F	T				
T	F	F				
F	T	T				
F	T	F				
F	F	T				
F	F	F				



Question 10 [4 marks]

Let  $P(x)$  be the predicate “ $x$  is an even number” and  $Q(x)$  be the predicate “ $x$  is divisible by 2”.

(a) Write out in words:  $\forall x \in \mathbb{Z}(\neg Q(x) \rightarrow \neg P(x))$  [2]

(b) Write out the negation of (a) in words [2]

Question 11 [5 marks]

Find the derivatives of the following. Simplify where possible.

(a)  $y = x^{\sqrt{x}}$  [3]

(b)  $y = \sin(\sin x)$  [2]

Question 12 [5 marks]

Prove using mathematical induction that for any integer  $n \geq 1$ ,  $n^5 - n$  is divisible by 5. [5]

Question 13 [3 marks]

Find an equation of the tangent to the curve:  $y = e^x \cos x$ , at  $x = 0$

Question 14 [4 marks]

Compute:  $\lim_{x \rightarrow 1^+} \left( \frac{1}{\ln x} - \frac{1}{x-1} \right)$ . (Use l'Hospital's rule if necessary.)

Question 15 [5 marks]

(a) Use part 1 of the Fundamental theorem of calculus to find  $g'(x)$  given that:

$$g(x) = \int_{1-2x}^{1+2x} t \sin t \, dt \quad [3]$$

(b) Use part 2 of the same theorem to evaluate  $\int_1^5 \frac{1}{x} \, dx$ . [2]

Question 16 [3 marks]

Given  $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$ , prove that:  $\lim_{\theta \rightarrow 0} \frac{\cos \theta - 1}{\theta} = 0$ .

Extra worksheet.

