UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF PURE AND APPLIED MATHEMATICS

MODULE: ASMA1A1

COURSE: CALCULUS OF ONE VARIABLE FUNCTIONS

(ALTERNATIVE SEMESTER)

CAMPUS: APK

SUPPLEMENTARY EXAM: JANUARY 2019

ASSESSOR:	MS ML JUGA
INTERNAL MODERATOR:	DR A CRAIG
DURATION: 2 HOURS	MARKS: 70

SURNAME AND INITIALS

STUDENT NUMBER

CONTACT NUMBER

NUMBER OF PAGES: 15 PAGES (including front page)
INSTRUCTIONS: ANSWER ALL THE QUESTIONS IN PEN
NO CALCULATORS ALLOWED.

If you require extra space, continue on the adjacent blank page next to it and indicate this clearly

Question 1 [8 marks]

For questions 1.1 - 1.8, choose **one** correct answer, and make a cross (X) in the correct block.

Question	a	b	c	d	e
1.1					
1.2					
1.3					
1.4					
1.5					
1.6					
1.7					
1.8					

1.1 Solve for x in the equation: $|4x-3|=2$. [1]

(a) $x=-\frac{1}{4}$ or $x=-\frac{5}{4}$

(b) $x=\frac{1}{4}$ or $x=\frac{5}{4}$

(c) $x=\frac{1}{4}$ or $x=-\frac{5}{4}$

(d) $x=-\frac{1}{4}$ or $x=\frac{5}{4}$

(e) None of the above

1.2 $\cot^2\theta =$ [1]

(a) $\csc^2\theta+1$

(b) $\csc^2\theta-1$

(c) $\tan^2\theta$

(d) $1-\sin^2\theta$

(e) None of the above

1.3 The correct expansion of $\sum_{k=1}^{5}(-1)^k \frac{3^k}{2^k}$ is: [1]

(a) $-\frac{3}{2}+\frac{3^2}{2^2}-\frac{3^3}{2^3}+\frac{3^3}{2^4}-\frac{3^5}{2^5}$

(b) $\frac{3}{2}-\frac{3^2}{2^2}+\frac{3^3}{2^3}-\frac{3^3}{2^4}+\frac{3^5}{2^5}$

(c) $-\frac{3}{2}-\frac{3^2}{2^2}-\frac{3^3}{2^3}-\frac{3^3}{2^4}-\frac{3^5}{2^5}$

(d) $\frac{3}{2}+\frac{3^2}{2^2}-\frac{3^3}{2^3}+\frac{3^3}{2^4}-\frac{3^5}{2^5}$

(e) None of the above

1.4 If $f(x) = x^4 - 1$, $g(x) = \sqrt[4]{x^2 - 2}$ and $h(x) = \sqrt{x+2}$, then $(f \circ g \circ h)(x)$ equals: [1]

(a) x^2

(b) x

(c) $2x$

(d) $x - 1$

(e) None of the above

1.5 solving $|3x - 4| < 1$ yields: [1]

(a) $x \leq 1$ or $x \geq \frac{5}{3}$

(b) $1 < x < \frac{5}{3}$

(c) $x < -1$ or $x > \frac{-5}{3}$

(d) $x \geq 1$ or $x \leq \frac{5}{3}$

(e) None of the above

1.6 $\lim_{x\to 0^+}(\sin ax + 1)^{\cot x}$ gives rise to the indeterminate form: [1]

(a) 1^0

(b) 0^0

(c) 1^∞

(d) 0^∞

(e) None of the above

1.7 The inverse of $\neg q \to p$ is: [1]

(a) $\neg q \wedge p$

(b) $\neg p \to q$

(c) $\neg q \to \neg p$

(d) $p \to q$

(e) None of the above

1.8 The derivative of $h(r) = \dfrac{ae^r}{b+e^r}$ is: [1]

(a) $\dfrac{abe^r}{(b+e^r)^2}$

(b) $\dfrac{abe^{2r}}{(a+e^r)^2}$

(c) $\dfrac{ae^r}{(b+e^r)^2}$

(d) $\dfrac{e^{2r}}{(b+e^r)^2}$

(e) None of the above

Question 2 [4 marks]

Solve for x if: $\dfrac{x^2}{x-1} \leq \dfrac{3x}{x-1} + \dfrac{10}{x-1}$

Question 3 [2 marks]

Sketch the graph of $y = \sec\theta$ for $\theta \in [0, 2\pi]$. Indicate clearly the intercepts and asymptotes if any.

Question 4 [8 marks]

Given the following case-defined function:

$$f(x) = \begin{cases} e^x + 1 & \text{if } x > 0 \\ -x^2 + 2 & \text{if } x \leq 0 \end{cases}$$

4.1 Graph the function [2]

4.2 Determine:

4.2.1 $\lim_{x \to 0^-} f(x)$ [1]

4.2.2 $\lim_{x \to 0^+} f(x)$ [1]

4.2.3 $\lim_{x \to 0} f(x)$ [1]

4.2.4 $f(0)$ [1]

4.3 Is f continuous at $x = 0$? Explain. [2]

Question 5 [4 marks]

If g is the function defined by $g(x) = -\ln(x-1)+1$

5.1 Sketch the graph of f by making use of translations. Do a separate sketch for each transformation, indicating the intercepts and asymptotes if any. [2]

5.2 State the domain. [1]

5.3 State the range. [1]

Question 6 [2 marks]

If f is the function defined below, determine whether f is odd even or neither:

$f(x) = -3 - 2\sin x$

Question 7 [3 marks]

If $\tan\theta = -\sqrt{3}$ and $\pi < \theta < 2\pi$, find the other five trigonometric ratios.

Question 8 [4 marks]

Determine:

(a) $\lim_{x \to -3} \dfrac{x^3 - 27}{x - 3}$ [2]

(b) $\lim_{x \to \infty} \dfrac{\sqrt{9x^8 + x + 10}}{3x^4 + 5x}$ [2]

Question 9 [3 marks]

If $n(x) = \sqrt{x}$, determine $n'(x)$ by making use of first principles.

Question 10 [3 marks]

Given the propositional formula $\neg p \rightarrow \neg q$:

(a) Give the contrapositive. [1]

(b) Give the converse. [1]

(c) Rewrite the proposition using only the $\vee$, $\wedge$, or $\neg$ connectives. [1]

Question 11 [10 marks]

(a) Prove using mathematical induction that for any integer $n \geq 1$, $n^5 - n$ is divisible by 5. [5]

(b) Prove by contraposition: If $m^2 + n^2 = 0$, then $n = 0$ and $m = 0$ [2]

(c) Is the following statement true or false? Hence prove it or disprove by a counter-example:

$\forall n \in \mathbb{Z}$, $n \geq 1$, $m \geq 1$, if m is odd and n is even, then $m + n$ is divisible by 3. [3]

Question 12 [2 marks]

Prove the following identity:

$$\tanh(\ln x) = \frac{x^2 - 1}{x^2 + 1}$$

Question 13 [5 marks]

Find the derivatives of the following. Simplify where possible.

(a) $y = \cosh x(1 + \ln \cosh x)$ [3]

(b) $y = \ln(e^{-x} + xe^{-x})$ [2]

Question 14 [3 marks]

Compute: $\lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{e^x - 1} \right)$. (Use L'Hospital's rule if necessary.)

Question 15 [5 marks]

(a) Use part 1 of the Fundamental Theorem of Calculus to find $g'(x)$ given that

$$g(x) = \int_{2x}^{3x} \frac{u^2 - 1}{u^2 + 1}\, du$$ [3]

(b) Use part 2 of the same theorem to evaluate $\int_{-1}^{1} e^{u+1}\, du$. [2]

Question 16 [4 marks]

State and prove the Product Rule of differentiation.

Extra worksheet.