

**UNIVERSITY OF JOHANNESBURG**

**FACULTY OF SCIENCE**



**DEPARTMENT OF PURE AND APPLIED MATHEMATICS**

**MODULE: ASMA1A1**

**COURSE: CALCULUS OF ONE VARIABLE FUNCTIONS**

**(ALTERNATIVE SEMESTER)**

**CAMPUS: APK**

**EXAM: NOVEMBER 2018**

**DATE: 19 NOVEMBER 2018**

**ASSESSOR: MS ML JUGA**

**INTERNAL MODERATOR: DR A CRAIG**

**DURATION: 2 HOURS MARKS: 70**

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**SURNAME AND INITIALS**

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**STUDENT NUMBER**

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**CONTACT NUMBER**

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**NUMBER OF PAGES: 13 PAGES (including front page)**

**INSTRUCTIONS: ANSWER ALL THE QUESTIONS IN PEN**

**NO CALCULATORS ALLOWED.**

**If you require extra space, continue on the adjacent blank page next to it and indicate this clearly**

Question 1 [9 marks]

For questions 1.1 - 1.9, choose **one** correct answer, and make a cross (X) in the correct block.

Question	a	b	c	d	e
1.1					
1.2					
1.3					
1.4					
1.5					
1.6					
1.7					
1.8					
1.9					

1.1 Which of the following is not an exact ratio for  $\frac{2\pi}{3}$ ? [1]

(a)  $\sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2}$

(b)  $\cot \frac{2\pi}{3} = \frac{1}{\sqrt{3}}$

(c)  $\csc \frac{2\pi}{3} = \frac{2}{\sqrt{3}}$

(d)  $\sec \frac{2\pi}{3} = -2$

(e) None of the above

1.2 Which of the following is not an identity of  $\cos 2x$ ? [1]

(a)  $2 \cos^2 x + 1$

(b)  $\cos^2 x - \sin^2 x$

(c)  $1 - 2 \sin^2 x$

(d)  $-\ln e + 2 \cos^2 x$

(e) None of the above

1.3 The correct expansion of  $\sum_{i=1}^3 (-1)^{i+1} \frac{3^i}{i+1}$  is:

[1]

(a)  $-\frac{3}{2} + \frac{3^2}{3} - \frac{3^3}{4}$

(b)  $\frac{3}{2} + \frac{3^2}{3} + \frac{3^3}{4}$

(c)  $\frac{3}{2} - \frac{3^3}{4} + \frac{3^4}{5}$

(d)  $-\frac{3}{2} - \frac{3^2}{3} - \frac{3^3}{4}$

(e) None of the above

1.4 If  $f(x) = x^3 - 1$ ,  $g(x) = \sqrt[3]{x^2 - 1}$  and  $h(x) = \sqrt{x + 2}$ , then  $(f \circ g \circ h)(x)$  equals:

[1]

(a)  $\sqrt{x}$

(b)  $x$

(c)  $\sqrt{x + 1}$

(d)  $x + 1$

(e) None of the above

1.5 Solving  $|7x + 1| > 2$  yields:

[1]

(a)  $x > -\frac{2}{7}$  or  $x < \frac{1}{7}$

(b)  $-\frac{2}{7} \leq x \leq \frac{1}{7}$

(c)  $x < -\frac{2}{7}$  or  $x > \frac{1}{7}$

(d)  $x \leq -\frac{2}{7}$  or  $x > \frac{1}{7}$

(e) None of the above

1.6 Which of the following is not a proposition? [1]

- (a) UJ is a university
- (b) Durban is the capital of South Africa
- (c) 8 minus 5 equals 10
- (d)  $x$  is an integer
- (e) None of the above

1.7 The contrapositive of  $\neg p \rightarrow q$  is: [1]

- (a)  $q \rightarrow p$
- (b)  $p \rightarrow q$
- (c)  $\neg q \rightarrow \neg p$
- (d)  $\neg p \rightarrow \neg q$
- (e) None of the above

1.8 The derivative of  $y = e^{\ln x} + \ln e^x$  is: [1]

- (a) 1
- (b)  $e$
- (c)  $\ln x$
- (d) 2
- (e) None of the above

1.9 Evaluating  $\int_{-1}^1 \frac{\tan x}{1+x^2+x^4} dx$  yields: [1]

- (a)  $\pi$
- (b)  $-1$
- (c) 0
- (d) 1
- (e) None of the above

Question 2 [8 marks]

Given the following case-defined function:

$$f(x) = \begin{cases} \ln x & \text{if } x > 1 \\ x^2 - 1 & \text{if } x < 1 \end{cases}$$

2.1 Graph the function

[2]

2.2 Determine:

2.2.1  $\lim_{x \rightarrow 1^-} f(x)$

[1]

2.2.2  $\lim_{x \rightarrow 1^+} f(x)$

[1]

2.2.3  $\lim_{x \rightarrow 1} f(x)$

[1]

2.2.4  $f(1)$

[1]

2.3 Is  $f$  differentiable at  $x = 1$ ? Explain using your results from 2.2.

[2]

Question 3 [4 marks]

Solve for  $x$  if:  $\frac{x}{x^2 + 3x - 4} < -\frac{3}{x^2 + 3x - 4}$

Question 4 [2 marks]

If  $g$  is the function defined below, determine whether  $g$  is odd, even or neither:

$$g(x) = 2 \sin x - 5 \cos x$$

Question 5 [2 marks]

Sketch the graph of  $y = \csc \theta$  for  $\theta \in (0, 2\pi)$ . Include all intercepts and asymptotes if any.

Question 6 [4 marks]

If  $j$  is the function defined by  $j(x) = 2e^{(x-1)} + 1$

5.1 Sketch the graph of  $f$  by making use of translations. Do a separate sketch for each transformation, showing clearly all intercepts. [2]

5.2 State the domain. [1]

5.3 State the range [1]

Question 7 [3 marks]

If  $m(x) = \frac{x+2}{x-2}$ , determine  $m'(x)$  by making use of first principles.

Question 8 [4 marks]

Determine:

(a)  $\lim_{x \rightarrow 1} \frac{x^4 - 1}{x - 1}$  [2]

(b)  $\lim_{x \rightarrow \infty} \frac{\sqrt{9x^4 - x}}{x^2 + 3}$  [2]

Question 9 [5 marks]

By completing the truth table below, Determine whether or not the following propositions are logically equivalent:

$p \rightarrow (q \rightarrow r)$  and  $(p \rightarrow q) \rightarrow r$

<b>p</b>	<b>q</b>	<b>r</b>				
T	T	T				
T	T	F				
T	F	T				
T	F	F				
F	T	T				
F	T	F				
F	F	T				
F	F	F				

Question 10 [4 marks]

Let  $P(x)$  be the predicate “ $x$  is an even number” and  $Q(x)$  be the predicate “ $x$  is divisible by 2”.

(a) Write out in words:  $\forall x \in \mathbb{Z}(\neg Q(x) \rightarrow \neg P(x))$  [2]

(b) Write out the negation of (a) in words [2]

Question 11 [5 marks]

Find the derivatives of the following. Simplify where possible.

(a)  $y = x^{\sqrt{x}}$  [3]

(b)  $y = \sin(\sin x)$  [2]

Question 12 [5 marks]

Prove using mathematical induction that for any integer  $n \geq 1$ ,  $n^5 - n$  is divisible by 5. [5]

Question 13 [3 marks]

Find an equation of the tangent to the curve:  $y = e^x \cos x$ , at  $x = 0$

Question 14 [4 marks]

Compute:  $\lim_{x \rightarrow 1^+} \left( \frac{1}{\ln x} - \frac{1}{x-1} \right)$ . (Use l'Hospital's rule if necessary.)

Question 15 [5 marks]

- (a) Use part 1 of the Fundamental theorem of calculus to find  $g'(x)$  given that:

$$g(x) = \int_{1-2x}^{1+2x} t \sin t \, dt \quad [3]$$

- (b) Use part 2 of the same theorem to evaluate  $\int_1^5 \frac{1}{x} \, dx$ . [2]

Question 16 [3 marks]

Given  $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$ , prove that:  $\lim_{\theta \rightarrow 0} \frac{\cos \theta - 1}{\theta} = 0$ .

Extra worksheet.

