UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF APPLIED MATHEMATICS

MODULE **APM03B3**
Multilinear Algebra

CAMPUS **APK**

EXAM **NOVEMBER 2018**

DATE: 20/11/2018 **SESSION:** 8:30–11:30

ASSESSOR Prof. W.-H Steeb

EXTERNAL MODERATOR Prof. Y. Hardy

DURATION: 3 HOURS **MARKS:** 30

NUMBER OF PAGES: 2 PAGES

INSTRUCTIONS: ANSWER ALL THE QUESTIONS

ALL CALCULATIONS MUST BE SHOWN

POCKET CALCULATORS ARE PERMITTED

ALL ANGLES ARE MEASURED IN RADIANS

THE PRESCRIBED TEXT BOOKS ARE ALLOWED

QUESTION 1

Consider the 4×4 matrix

$$A = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}.$$

(i) Find the determinant of A and thus show that A has an inverse.
(ii) Calculate the inverse of A applying the Cayley-Hamilton theorem.
(iii) Calculate the inverse of A applying the spectral theorem. Of course first check whether the spectral theorem can be applied.
(iv) Can the matrix A be written as a Kronecker product of two 2×2 matrices? Prove or disprove. **(10)**

QUESTION 2

Find the singular value decomposition of

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \quad \text{and} \quad A \otimes A.$$

(10)

QUESTION 3

Let $\phi \in \mathbb{R}$. Consider the 4×4 unitary matrix

$$U(\phi) = \begin{pmatrix} 0 & 0 & 0 & e^{i\phi} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$

(i) Find the eigenvalues and normalized eigenvectors of $U(\phi)$. Do the eigenvalues depend on ϕ? Do the normalized eigenvectors depend on ϕ?
(ii) Let $\phi = \pi$. Find the group generated by $U(\pi)$ under matrix multiplication.
(iii) Can the matrix be written as the Kronecker product of two 2×2 matrices? Prove or disprove. **(10)**

END OF QUESTION PAPER