UNIVERSITY OF JOHANNESBURG

SM	
EM	
FM	

FACULTY OF SCIENCE

DEPARTMENT OF APPLIED PHYSICS AND ENGINEERING MATHEMATICS

BACHELOR OF ENGINEERING TECHNOLOGY:
CHEMICAL/CIVIL/ELECTRICAL/INDUSTRIAL/MECHANICAL

MODULE MATE2A2

ENGINEERING MATHEMATICS V 2A

CAMPUS **DFC**

JUNE EXAMINATION

DATE: 02 JUNE 2018 **SESSION: 12:30 – 15:30**

ASSESSORS: **DR PG DLAMINI, MR IK LETLHAGE, DR SM SIMELANE**

INTERNAL MODERATOR: **MR T PAEPAE**

DURATION: 3 HOURS **MARKS: 100**

SURNAME AND INITIALS: _____

STUDENT NUMBER: _____

CONTACT NUMBER: _____

LECTURER	QUALIFICATION	MARK (X)
DR PG DLAMINI	ELECTRICAL ENGINEERING	
MR IK LETLHAGE	INDUSTRIAL AND MECHANICAL ENGINEERING	
DR SM SIMELANE	CHEMICAL AND CIVIL ENGINEERING	

NUMBER OF PAGES: 18 (INCLUDING COVER PAGE)

REQUIREMENTS : NON-PROGRAMMABLE SCIENTIFIC CALCULATOR

INSTRUCTIONS : ANSWER ALL THE QUESTIONS
USE THE BLANK PAGES AT THE BACK TO DO ROUGH WORK
NO PAGES SHOULD BE REMOVED FROM THIS PAPER.
USE ONLY BLUE OR BLACK INK TO WRITE. NO PENCIL.

QUESTION 1

(a) Newton's law of cooling states that a body of uniform composition, when placed in an environment with a constant temperature, will approach the temperature of the environment at a rate that is proportional to the difference in temperature of the body (T) and the temperature of the environment (T_a). This law is modelled by the differential equation:

$$\frac{dT}{dt} = k(T - T_a)$$

(i) A chicken has been in the refrigerator for several days and has a uniform temperature of 40°F. An oven is preheated to 325 °F. The chicken is placed in the oven for 20 minutes and when taken out its temperature is found to be 60 °F.

Solve the differential equation and express the temperature T in terms of time t. (5)

(ii) Approximately how long does the chicken have to stay in the oven to have a temperature of 185 °F? (2)

(b) Identify and solve the differential equation: $\frac{dy}{dx}+\frac{1}{x}y=3x^2y^3$ (6)

(c) Solve the initial value problem $x^2 \frac{dy}{dx} + xy \frac{dy}{dx} = y^2, \quad y(1) = 1.$ (5)

QUESTION 2

Let $f(t)=\begin{cases} t^2 & 0 \le t < 2 \\ t-1 & 2 \le t < 4 \\ -3 & t \ge 4 \end{cases}$

(a) Sketch the graph of the given function. (3)

(b) Express the function $f(t)$ in terms of Heaviside unit step functions (2)

QUESTION 3

Evaluate:

(a) $L\{\sin(t-\pi)H(t-2\pi)\}$ (3)

(b) $L^{-1}\left\{\dfrac{s^2+2k^2}{s(s^2+4k^2)}\right\}$ (5)

QUESTION 4

Use **D-operators** to find the general solution of the ordinary differential equation $3y'' - 5y' = x\,e^{-x}$. (8)

QUESTION 5

A series circuit contains a resistor with resistance $R = 20\Omega$, an inductor with inductance $L = 1H$, a capacitor with capacitance $C = 0.01F$ and an external force $E(t) = 12\sin 10t$. The charge q can be modelled by the ordinary differential equation $L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = E(t)$, with initial conditions $q(0) = 0,\ q'(0) = 1$.

(a) Use **Laplace transforms** to solve the equation for q. (7)

(b) Use **D-operators** to solve the equation for q and use the given initial conditions to determine the particular solution. (8)

QUESTION 6

Use Laplace transforms to solve the following initial value problems.

(a) $y''+2y'-3y=e^{-3(t-2)}H(t-2),\ y(0)=1,\ y'(0)=1.$ (9)

(b) $\frac{d^2y}{dt^2} - 4\frac{dy}{dt} - 6y = 0$ subject to $y(0) = -1$ and $y'(0) = 0$. (5)

QUESTION 7

Use **Laplace transforms** to solve the following system of differential equations for x **only**.

$$2x - y - \frac{dy}{dt} = 4e^{-t}$$

$$2\frac{dx}{dt} + y = 2$$

$$x(0) = y(0) = 0$$ (12)

QUESTION 8

Let $g(x)=\begin{cases} x-1 & \text{when } -1\le x<0 \\ x+1 & \text{when } 0\le x<1 \end{cases}$

Determine if g is even, odd or neither. (2)

QUESTION 9

Find the half-range Fourier cosine series for the periodic function

$$f(x)=\begin{cases} x, & 0\le x<1 \\ 0, & 1\le x<2 \end{cases}$$ (6)

if $f(x)=f(x+4)$.

QUESTION 10

Given the series for the non-periodic function $g(t) = 2t$ over $-\pi \leq t \leq \pi$ as

$$2x = 4(\sin x - \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x - \frac{1}{4}\sin 4x + \frac{1}{5}\sin 5x - \frac{1}{6}\sin 6x + \cdots).$$

Let $x = \frac{\pi}{2}$ and deduce a series for $\frac{\pi}{4}$. (2)

QUESTION 11

Determine the complex Fourier series for the function

$$f(x) = \begin{cases} 0, & -2 \leq x < -1 \\ 5, & -1 \leq x < 1 \\ 0, & 1 \leq x < 2 \end{cases} \qquad (5)$$

The function is periodic outside this range of period 4.

QUESTION 12

The table below defines a discrete function $f(x)$ of period 360^0. Determine the Fourier series of $f(x)$ up to the second harmonic. (Work to 2 decimal places) (5)

x^0	0	30	60	90	120	150	180	210	240	270	300	330
$f(x)$	0	8,0	11,5	6	4	5,4	0	-8	-11,5	-6	-4	-5,4

End of assessment – Total 100 marks available

Use this space if you want to redo any question(s). Please indicate clearly at the relevant question(s) that the solution is on this page.