UNIVERSITY OF JOHANNESBURG

SM	
EM	
FM	

FACULTY OF SCIENCE

DEPARTMENT OF APPLIED PHYSICS AND ENGINEERING MATHEMATICS

NATIONAL DIPLOMA:

CHEMICAL ENGINEERING, ENGINEERING/EXTRACTION METALLURGY

MODULE: **MAT2AE2 ☐ /ASMA2E2 ☐ (Please mark correct code)**
ENGINEERING MATHEMATICS 2

CAMPUS: **DFC**

JULY EXAMINATION

DATE: **16 JULY 2018** **SESSION: 11:30 – 13:30**

DURATION: **2 HOURS** **MARKS: 65**

ASSESSOR: **PG DLAMINI**

MODERATOR: **IK LETLHAGE**

INITIALS AND SURNAME: ______________________________

STUDENT NUMBER: ______________________________

CONTACT NUMBER: ______________________________

NUMBER OF PAGES: **13 (EXCLUDING COVER PAGE)**

INSTRUCTIONS: **ANSWER ALL QUESTIONS IN THE SPACES PROVIDED.**
USE THE BACK OF EACH PAGE FOR ROUGH WORK
USE ONLY A PEN FOR WRITING AND DRAWING (BLACK OR BLUE).

REQUIREMENTS: **NON PROGRAMMABLE CALCULATOR.**
FORMULA SHEET (PROVIDED).

SECTION A [10]

INSTRUCTIONS

USE THE TABLE ON PAGE 2 TO MARK THE LETTER (X) CORRESPONDING TO THE CORRECT ANSWER. DO YOUR ROUGH WORK ON THE BLANK PAGES.

1. $\frac{d}{dx}\left(\tan^{-1}\frac{1}{x}\right) =$ (2)

A $\frac{x^2}{x^2+1}$

B $-\frac{1}{x^2}\left(\sec^{-1}\frac{1}{x}\right)^2$

C $-\frac{1}{x^2+1}$

D none of these

2. If $y = \ln(\ln(\sin x^2))$ then $\frac{dy}{dx} =$ (2)

A $\frac{2x}{\sin x^2}$

B $\frac{2x \cot x^2}{\ln(\sin x^2)}$

C $\frac{2x \cos x}{\ln(\sin x^2)}$

D none of these

3. If $z = \sin 2x \cos y$, then $\frac{\partial^2 z}{\partial x^2}$ at the point $(2,1)$ is equal to: (2)

A 1,64

B -0,71

C 0.41

D none of these

4. $\int_0^4 \frac{\left(1+\sqrt{x}\right)^2}{\sqrt{x}}\,dx =$ (2)

A $\frac{4}{3}$

B $\frac{26}{3}$

C $\frac{52}{3}$

D none of these

5. $\int \frac{x}{\sqrt{16-x^4}}\,dx =$ (2)

A $\frac{1}{2}\sin^{-1}(4x^2)+C$

B $\frac{1}{2}\sin^{-1}\left(\frac{x^2}{4}\right)+C$

C $\frac{1}{4}\sin^{-1}\left(\frac{x^2}{2}\right)+C$

D none of these

1.	A	B	C	D
2.	A	B	C	D
3.	A	B	C	D
4.	A	B	C	D
5.	A	B	C	D

SECTION B [55 MARKS]

INSTRUCTIONS

SHOW ALL THE STEPS TAKEN AND GIVE YOUR FINAL ANSWER CORRECT TO TWO DECIMAL PLACES, WHERE APPLICABLE. SIMPLIFY YOUR ANSWERS FULLY.

6. Given $y = \dfrac{\cos t\sqrt{\ln t}}{t^2}$, find $\dfrac{dy}{dt}$ (4)

7. If $2x^2 - 3y^2 = 4$, show that $\dfrac{d^2y}{dx^2} = \dfrac{-8}{9y^3}$. (4)

8. If $z = \dfrac{xy}{x+y}$, show that

$$x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = 0$$ (6)

9. Find $\dfrac{d^2y}{dx^2}$ if $y = \dfrac{t^2}{t-1}$ and $x = \dfrac{1}{t-1}$ (5)

10. A right angled triangle has shorter sides of length $a = 3$cm and $b = 4$ cm. The hypotenuse is given by $c = (a^2 + b^2)^{\frac{1}{2}}$. Calculate the change to the hypotenuse if a is increased by 0.1 cm and b is decreased by 0.2 cm. (5)

11. Integrate the following,

11.1. $\int \frac{dx}{x\sqrt{1-(\ln x)^2}}$ (2)

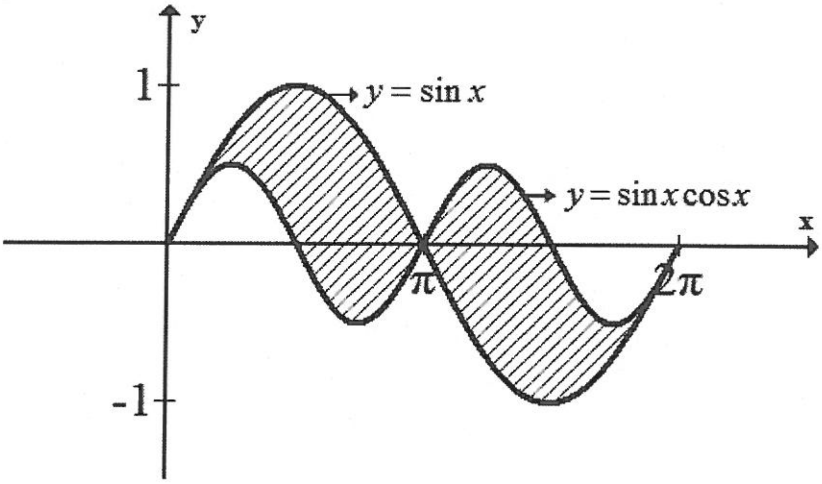
11.2. $\int \frac{\ln x}{x^2}\,dx$ (4)

11.3. $\int \frac{3x+2}{x^2+4x+16}\,dx$ (5)

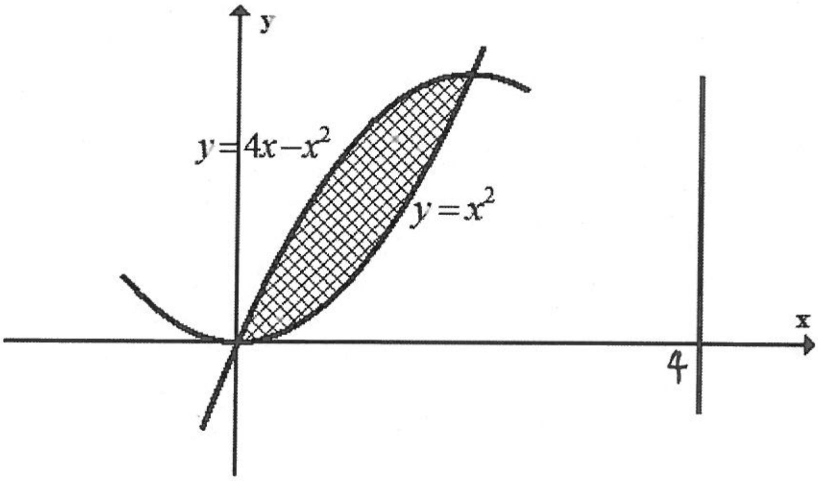
11.4. $\int \sin^4 3t \; dt$ (4)

11.5. $\int \frac{x^2 - 2x - 5}{x^3 - 5x^2} dx$ (6)

12. The graphs of $y = \sin x$ and $y = \sin x \cos x$ are shown in the figure below. Calculate the area enclosed by the two graphs (i.e. the shaded area). (5)



13. Determine the volume of the solid generated by revolving the area bounded by $y = x^2$ and $y = 4x - x^2$ about the line $x = 4$. (See figure below) (5)



End of assessment – Total 65 marks

Use this space if you want to redo any question(s). Please indicate clearly at the relevant question(s) that the solution is on this page.

TRIGONOMETRY

DEFINITIONS

$$\sin\theta = \frac{y}{r} = \frac{1}{\operatorname{cosec}\theta}$$

$$\cos\theta = \frac{x}{r} = \frac{1}{\sec\theta}$$

$$\tan\theta = \frac{y}{x} = \frac{1}{\cot\theta}$$

IDENTITIES

$$\sin^2\theta + \cos^2\theta = 1$$
$$1 + \tan^2\theta = \sec^2\theta$$
$$\cot^2\theta + 1 = \operatorname{cosec}^2\theta$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\sin(-\theta) = -\sin\theta$$
$$\cos(-\theta) = +\cos\theta$$
$$\tan(-\theta) = -\tan\theta$$

COMPOUND ANGLES

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$
$$\sin(A-B) = \sin A\cos B - \cos A\sin B$$
$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$
$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$
$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$
$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

DOUBLE ANGLES

$$\sin 2A = 2\sin A\cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\sin^2 A = \tfrac{1}{2}(1 - \cos 2A)$$
$$\cos^2 A = \tfrac{1}{2}(1 + \cos 2A)$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

PRODUCTS OF SINES AND COSINES

$$\sin A\cos B = \tfrac{1}{2}[\sin(A+B) + \sin(A-B)]$$
$$\cos A\sin B = \tfrac{1}{2}[\sin(A+B) - \sin(A-B)]$$
$$\cos A\cos B = \tfrac{1}{2}[\cos(A+B) + \cos(A-B)]$$
$$\sin A\sin B = +\tfrac{1}{2}[\cos(A-B) - \cos(A+B)]$$

HYPERBOLIC FUNCTIONS

IDENTITIES

$$\cosh^2 x - \sinh^2 x = 1$$
$$1 - \tanh^2 x = \operatorname{sech}^2 x$$
$$\coth^2 x - 1 = \operatorname{cosech}^2 x$$
$$\sinh^2 x = \tfrac{1}{2}(\cosh 2x - 1)$$
$$\cosh^2 x = \tfrac{1}{2}(\cosh 2x + 1)$$
$$\sinh 2x = 2\sinh x\cosh x$$
$$\cosh 2x = \cosh^2 x + \sinh^2 x$$
$$= 2\cosh^2 x - 1$$
$$= 1 + 2\sinh^2 x$$

DEFINITIONS

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\operatorname{cosech} x = \frac{1}{\sinh x}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\operatorname{sech} x = \frac{1}{\cosh x}$$

$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\coth x = \frac{1}{\tanh x}$$

RULES OF DIFFERENTIATION

1. $\frac{d}{dx}(C) = 0$

2. $\frac{d}{dx}(ax^n) = nax^{n-1}$

3. If $y = f(x) \pm g(x) \pm h(x)$ then
$\frac{dy}{dx} = f'(x) \pm g'(x) \pm h'(x)$

4. $\frac{d}{dx}[f(x)]^n = n[f(x)]^{n-1}.f'(x)$

5. $\frac{d}{dx}\{f(x).g(x)\} = f(x).g'(x) + f'(x).g(x)$

6. $\frac{d}{dx}\left\{\frac{f(x)}{g(x)}\right\} = \frac{g(x).f'(x) - f(x).g'(x)}{[g(x)]^2}$

7. $\frac{d}{dx}\left\{a^{f(x)}\right\} = a^{f(x)}.f'(x).\lambda n\, a$

8. $\frac{d}{dx}\left\{e^{f(x)}\right\} = e^{f(x)}.f'(x)$

9. $\frac{d}{dx}\{\log_a f(x)\} = \frac{1}{f(x)}.\log_a e.f'(x)$

10. $\frac{d}{dx}[\ln f(x)] = \frac{1}{f(x)}.\ f'(x)$

11. $\frac{d}{dx}\{\sin f(x)\} = f'(x)\cos f(x)$

12. $\frac{d}{dx}\{\cos f(x)\} = -f'(x)\sin f(x)$

13. $\frac{d}{dx}\{\tan f(x)\} = f'(x)\sec^2 f(x)$

14. $\frac{d}{dx}\{\text{cosec} f(x)\} = -f'(x)\text{cosec} f(x)\cot f(x)$

15. $\frac{d}{dx}\{\sec f(x)\} = f'(x)\sec f(x)\tan f(x)$

16. $\frac{d}{dx}\{\cot f(x)\} = -f'(x)\text{cosec}^2 f(x)$

17. $\frac{d}{dx}\{\sinh f(x)\} = f'(x)\cosh f(x)$

18. $\frac{d}{dx}\{\cosh f(x)\} = f'(x)\sinh f(x)$

19. $\frac{d}{dx}\{\tanh f(x)\} = f'(x)\text{sech}^2 f(x)$

20. $\frac{d}{dx}\{\text{cosech} f(x)\} = -f'(x)\text{cosech} f(x)\coth f(x)$

21. $\frac{d}{dx}\{\text{sech} f(x)\} = -f'(x)\text{sech} f(x)\tanh f(x)$

22. $\frac{d}{dx}\{\coth f(x)\} = -f'(x)\text{cosech}^2 f(x)$

23. $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dv} \times \frac{dv}{dx}$

24. **Parametric Equations**

25. If $y = f(t)$ and $x = g(t)$, then

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \qquad \frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$$

26. $\frac{d}{dx}\left(\sin^{-1} f(x)\right) = \frac{f'(x)}{\sqrt{1-[f(x)]^2}}$

27. $\frac{d}{dx}\left(\cos^{-1} f(x)\right) = \frac{-f'(x)}{\sqrt{1-[f(x)]^2}}$

28. $\frac{d}{dx}\left(\tan^{-1} f(x)\right) = \frac{f'(x)}{1+[f(x)]^2}$

29. $\frac{d}{dx}\left(\cot^{-1} f(x)\right) = \frac{-f'(x)}{1+[f(x)]^2}$

30. $\frac{d}{dx}\left(\sec^{-1} f(x)\right) = \frac{f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$

31. $\frac{d}{dx}\left(\text{cosec}^{-1} f(x)\right) = \frac{-f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$

32. $\frac{d}{dx}\left(\sinh^{-1} f(x)\right) = \frac{f'(x)}{\sqrt{[f(x)]^2+1}}$

33. $\frac{d}{dx}\left(\cosh^{-1} f(x)\right) = \frac{f'(x)}{\sqrt{[f(x)]^2-1}}$

34. $\frac{d}{dx}\left(\tanh^{-1} f(x)\right) = \frac{f'(x)}{1-[f(x)]^2}$

35. $\frac{d}{dx}\left(\coth^{-1} f(x)\right) = \frac{f'(x)}{1-[f(x)]^2}$

36. $\frac{d}{dx}\left(\text{sech}^{-1} f(x)\right) = \frac{-f'(x)}{f(x)\sqrt{1-[f(x)]^2}}$

37. $\frac{d}{dx}\left(\text{cosech}^{-1} f(x)\right) = \frac{-f'(x)}{|f(x)|\sqrt{[f(x)]^2+1}}$

38. **Small Increments**
If $z = f(x, y, w)$; then
$\delta z = \frac{\partial z}{\partial x}\delta x + \frac{\partial z}{\partial y}\delta y + \frac{\partial z}{\partial w}\delta w$

39. **Rates of change**
If $z = f(x, y, w)$; then
$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt} + \frac{\partial z}{\partial w}\frac{dw}{dt}$

STANDARD INTEGRALS

1. $\int f'(x)[f(x)]^n\,dx = \dfrac{1}{n+1}[f(x)]^{n+1} + C \quad (n \neq -1)$

2. $\int \dfrac{f'(x)}{f(x)}\,dx = \lambda n|f(x)| + C$

3. $\int f'(x)a^{f(x)}dx = \dfrac{1}{\ln a}a^{f(x)} + C$ (*a* being constant)

4. $\int f'(x)e^{f(x)}dx = e^{f(x)} + C$

5. $\int f'(x)\sin f(x)dx = -\cos f(x) + C$

6. $\int f'(x)\cos f(x)dx = \sin f(x) + C$

7. $\int f'(x)\tan f(x)dx = \lambda n|\sec f(x)| + C$

8. $\int f'(x)\cot f(x)dx = \lambda n|\sin f(x)| + C$

9. $\int f'(x)\sec f(x)dx = \lambda n|\sec f(x) + \tan f(x)| + C$

10. $\int f'(x)\operatorname{cosec} f(x)dx = 1n|\operatorname{cosec} f(x) - \cot f(x)| + C$

11. $\int f'(x)\sec^2 f(x)dx = \tan f(x) + C$

12. $\int f'(x)\operatorname{cosec}^2 f(x)dx = -\cot f(x) + C$

13. $\int f'(x)\sec f(x)\tan f(x)dx = \sec f(x) + C$

14. $\int f'(x)\operatorname{cosec} f(x)\cot f(x)dx = -\operatorname{cosec} f(x) + C$

15. $\int f'(x)\sinh f(x)dx = \cosh f(x) + C$

16. $\int f'(x)\cosh f(x)dx = \sinh f(x) + C$

17. $\int f'(x)\tanh f(x)dx = \lambda n\cosh f(x) + C$

18. $\int f'(x)\coth f(x)dx = \lambda n\sinh f(x) + C$

19. $\int f'(x)\operatorname{sech}^2 f(x)dx = \tanh f(x) + C$

20. $\int f'(x)\operatorname{cosech}^2 f(x)dx = -\coth f(x) + C$

21. $\int f'(x)\operatorname{sech} f(x)\tanh f(x)dx = -\operatorname{sech} f(x) + C$

22 $\int f'(x)\operatorname{cosech} f(x)\coth f(x)dx = -\operatorname{cosech} f(x) + C$

23. $$\int \frac{f'(x)}{[f(x)]^2 - a^2}dx = \frac{1}{2a}1n\left|\frac{f(x)-a}{f(x)+a}\right| + C = -\frac{1}{a}\coth^{-1}\frac{f(x)}{a} + C$$

24. $$\int \frac{f'(x)}{a^2 - [f(x)]^2}dx = \frac{1}{2a}1n\left|\frac{a+f(x)}{a-f(x)}\right| + C = \frac{1}{a}\tanh^{-1}\frac{f(x)}{a} + C$$

25. $$\int \frac{f'(x)}{[f(x)]^2 + a^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + C$$

26. $\int \frac{f'(x)}{\sqrt{a^2-[f(x)]^2}}dx = \sin^{-1}\frac{f(x)}{a}+C$

27. $\int \frac{f'(x)}{\sqrt{[f(x)]^2+a^2}}dx = \lambda n\left|f(x)+\sqrt{[f(x)]^2+a^2}\right|+C = \sinh^{-1}\frac{f(x)}{a}+C$

28. $\int \frac{f'(x)}{\sqrt{[f(x)]^2-a^2}}dx = \lambda n\left|f(x)+\sqrt{[f(x)]^2-a^2}\right|+C = \cosh^{-1}\frac{f(x)}{a}+C$

29. $\int f'(x)\sqrt{a^2-[f(x)]^2}\,dx = \frac{a^2}{2}\sin^{-1}\frac{f(x)}{a}+\frac{f(x)}{2}\sqrt{a^2-[f(x)]^2}+C$

30. $\int f'(x)\sqrt{[f(x)]^2+a^2}\,dx = \frac{a^2}{2}\lambda n\left|f(x)+\sqrt{[f(x)]^2+a^2}\right|+\frac{f(x)}{2}\sqrt{[f(x)]^2+a^2}+C$

31. $\int f'(x)\sqrt{[f(x)]^2-a^2}\,dx = -\frac{a^2}{2}\lambda n\left|f(x)+\sqrt{[f(x)]^2-a^2}\right|+\frac{f(x)}{2}\sqrt{[f(x)]^2-a^2}+C$

32. **Integration by parts**

$\int u\,dv = uv - \int v\,du$

Priorities for *u*: inverse functions; log functions; x^n; e^{kx}; others

33. **Fundamental theorem of integration**

$\int_a^b f(x)dx = [F(x)]_a^b = F(b)-F(a)$

34. Mean value $= \frac{1}{b-a}\int_a^b f(x)\,dx$

35. $(\text{R.M.S})^2 = \frac{1}{b-a}\int_a^b (f(x))^2 dx$

J. INTEGRATION APPLICATIONS

VOLUMES OF SOLIDS OF REVOLUTION

For a vertical strip selection:

1. Disc: $V = \pi\int_a^b y^2 dx$
2. Washer: $V = \pi\int_a^b (y_2{}^2 - y_1{}^2)dx$
3. Shell: $V = 2\pi\int_a^b xy\,dx$

TRIGONOMETRY

DEFINITIONS

$$\sin\theta = \frac{y}{r} = \frac{1}{\operatorname{cosec}\theta}$$

$$\cos\theta = \frac{x}{r} = \frac{1}{\sec\theta}$$

$$\tan\theta = \frac{y}{x} = \frac{1}{\cot\theta}$$

IDENTITIES

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$\cot^2\theta + 1 = \operatorname{cosec}^2\theta$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\sin(-\theta) = -\sin\theta$$

$$\cos(-\theta) = +\cos\theta$$

$$\tan(-\theta) = -\tan\theta$$

COMPOUND ANGLES

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

$$\sin(A-B) = \sin A\cos B - \cos A\sin B$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

DOUBLE ANGLES

$$\sin 2A = 2\sin A\cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\sin^2 A = \tfrac{1}{2}(1 - \cos 2A)$$

$$\cos^2 A = \tfrac{1}{2}(1 + \cos 2A)$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

PRODUCTS OF SINES AND COSINES

$$\sin A\cos B = \tfrac{1}{2}[\sin(A+B) + \sin(A-B)]$$

$$\cos A\sin B = \tfrac{1}{2}[\sin(A+B) - \sin(A-B)]$$

$$\cos A\cos B = \tfrac{1}{2}[\cos(A+B) + \cos(A-B)]$$

$$\sin A\sin B = +\tfrac{1}{2}[\cos(A-B) - \cos(A+B)]$$

HYPERBOLIC FUNCTIONS

IDENTITIES

$$\cosh^2 x - \sinh^2 x = 1$$

$$1 - \tanh^2 x = \operatorname{sech}^2 x$$

$$\coth^2 x - 1 = \operatorname{cosech}^2 x$$

$$\sinh^2 x = \tfrac{1}{2}(\cosh 2x - 1)$$

$$\cosh^2 x = \tfrac{1}{2}(\cosh 2x + 1)$$

$$\sinh 2x = 2\sinh x\cosh x$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

$$= 2\cosh^2 x - 1$$

$$= 1 + 2\sinh^2 x$$

DEFINITIONS

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\operatorname{cosech} x = \frac{1}{\sinh x}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\operatorname{sech} x = \frac{1}{\cosh x}$$

$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\coth x = \frac{1}{\tanh x}$$

RULES OF DIFFERENTIATION

1. $\frac{d}{dx}(C)=0$

2. $\frac{d}{dx}(ax^n)=nax^{n-1}$

3. If $y=f(x)\pm g(x)\pm h(x)$ then
$\frac{dy}{dx}=f'(x)\pm g'(x)\pm h'(x)$

4. $\frac{d}{dx}[f(x)]^n=n[f(x)]^{n-1}.f'(x)$

5. $\frac{d}{dx}\{f(x).g(x)\}=f(x).g'(x)+f'(x).g(x)$

6. $\frac{d}{dx}\left\{\frac{f(x)}{g(x)}\right\}=\frac{g(x).f'(x)-f(x).g'(x)}{[g(x)]^2}$

7. $\frac{d}{dx}\left\{a^{f(x)}\right\}=a^{f(x)}.f'(x).\lambda n\,a$

8. $\frac{d}{dx}\left\{e^{f(x)}\right\}=e^{f(x)}.f'(x)$

9. $\frac{d}{dx}\{\log_a f(x)\}=\frac{1}{f(x)}.\log_a e.f'(x)$

10. $\frac{d}{dx}[\ln f(x)]=\frac{1}{f(x)}.\ f'(x)$

11. $\frac{d}{dx}\{\sin f(x)\}=f'(x)\cos f(x)$

12. $\frac{d}{dx}\{\cos f(x)\}=-f'(x)\sin f(x)$

13. $\frac{d}{dx}\{\tan f(x)\}=f'(x)\sec^2 f(x)$

14. $\frac{d}{dx}\{\operatorname{cosec} f(x)\}=-f'(x)\operatorname{cosec} f(x)\cot f(x)$

15. $\frac{d}{dx}\{\sec f(x)\}=f'(x)\sec f(x)\tan f(x)$

16. $\frac{d}{dx}\{\cot f(x)\}=-f'(x)\operatorname{cosec}^2 f(x)$

17. $\frac{d}{dx}\{\sinh f(x)\}=f'(x)\cosh f(x)$

18. $\frac{d}{dx}\{\cosh f(x)\}=f'(x)\sinh f(x)$

19. $\frac{d}{dx}\{\tanh f(x)\}=f'(x)\operatorname{sech}^2 f(x)$

20. $\frac{d}{dx}\{\operatorname{cosech} f(x)\}=-f'(x)\operatorname{cosech} f(x)\coth f(x)$

21. $\frac{d}{dx}\{\operatorname{sech} f(x)\}=-f'(x)\operatorname{sech} f(x)\tanh f(x)$

22. $\frac{d}{dx}\{\coth f(x)\}=-f'(x)\operatorname{cosech}^2 f(x)$

23. $\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dv}\times\frac{dv}{dx}$

24. **Parametric Equations**

25. If $y=f(t)$ and $x=g(t)$, then

$$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}} \qquad \frac{d^2y}{dx^2}=\frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$$

26. $\frac{d}{dx}\left(\sin^{-1} f(x)\right)=\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$

27. $\frac{d}{dx}\left(\cos^{-1} f(x)\right)=\frac{-f'(x)}{\sqrt{1-[f(x)]^2}}$

28. $\frac{d}{dx}\left(\tan^{-1} f(x)\right)=\frac{f'(x)}{1+[f(x)]^2}$

29. $\frac{d}{dx}\left(\cot^{-1} f(x)\right)=\frac{-f'(x)}{1+[f(x)]^2}$

30. $\frac{d}{dx}\left(\sec^{-1} f(x)\right)=\frac{f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$

31. $\frac{d}{dx}\left(\operatorname{cosec}^{-1} f(x)\right)=\frac{-f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$

32. $\frac{d}{dx}\left(\sinh^{-1} f(x)\right)=\frac{f'(x)}{\sqrt{[f(x)]^2+1}}$

33. $\frac{d}{dx}\left(\cosh^{-1} f(x)\right)=\frac{f'(x)}{\sqrt{[f(x)]^2-1}}$

34. $\frac{d}{dx}\left(\tanh^{-1} f(x)\right)=\frac{f'(x)}{1-[f(x)]^2}$

35. $\frac{d}{dx}\left(\coth^{-1} f(x)\right)=\frac{f'(x)}{1-[f(x)]^2}$

36. $\frac{d}{dx}\left(\operatorname{sech}^{-1} f(x)\right)=\frac{-f'(x)}{f(x)\sqrt{1-[f(x)]^2}}$

37. $\frac{d}{dx}\left(\operatorname{cosech}^{-1} f(x)\right)=\frac{-f'(x)}{|f(x)|\sqrt{[f(x)]^2+1}}$

38. **Small Increments**

If $z=f(x,y,w)$; then

$$\delta z=\frac{\partial z}{\partial x}\delta x+\frac{\partial z}{\partial y}\delta y+\frac{\partial z}{\partial w}\delta w$$

39. **Rates of change**

If $z=f(x,y,w)$; then

$$\frac{dz}{dt}=\frac{\partial z}{\partial x}\frac{dx}{dt}+\frac{\partial z}{\partial y}\frac{dy}{dt}+\frac{\partial z}{\partial w}\frac{dw}{dt}$$

STANDARD INTEGRALS

1. $\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + C \quad (n \neq -1)$	12. $\int f'(x)\text{cosec}^2 f(x)dx = -\cot f(x) + C$
2. $\int \frac{f'(x)}{f(x)} dx = \lambda n\lvert f(x)\rvert + C$	13. $\int f'(x)\sec f(x)\tan f(x)dx = \sec f(x) + C$
3. $\int f'(x)a^{f(x)}dx = \frac{1}{\ln a}a^{f(x)} + C$ (a being constant)	14. $\int f'(x)\text{cosec } f(x)\cot f(x)dx = -\text{cosec } f(x) + C$
4. $\int f'(x)e^{f(x)}dx = e^{f(x)} + C$	15. $\int f'(x)\sinh f(x)dx = \cosh f(x) + C$
5. $\int f'(x)\sin f(x)dx = -\cos f(x) + C$	16. $\int f'(x)\cosh f(x)dx = \sinh f(x) + C$
6. $\int f'(x)\cos f(x)dx = \sin f(x) + C$	17. $\int f'(x)\tanh f(x)dx = \lambda n\cosh f(x) + C$
7. $\int f'(x)\tan f(x)dx = \lambda n\lvert\sec f(x)\rvert + C$	18. $\int f'(x)\coth f(x)dx = \lambda n\sinh f(x) + C$
8. $\int f'(x)\cot f(x)dx = \lambda n\lvert\sin f(x)\rvert + C$	19. $\int f'(x)\text{sech}^2 f(x)dx = \tanh f(x) + C$
9. $\int f'(x)\sec f(x)dx = \lambda n\lvert\sec f(x) + \tan f(x)\rvert + C$	20. $\int f'(x)\text{cosech}^2 f(x)dx = -\coth f(x) + C$
10. $\int f'(x)\text{cosec} f(x)dx = 1n\lvert\text{cosec} f(x) - \cot f(x)\rvert + C$	21. $\int f'(x)\text{sech } f(x)\tanh f(x)dx = -\text{sech } f(x) + C$
11. $\int f'(x)\sec^2 f(x)dx = \tan f(x) + C$	22 $\int f'(x)\text{cosech} f(x)\coth f(x)dx = -\text{cosech} f(x) + C$

23. $$\int \frac{f'(x)}{[f(x)]^2 - a^2}dx = \frac{1}{2a}1n\left|\frac{f(x)-a}{f(x)+a}\right| + C = -\frac{1}{a}\coth^{-1}\frac{f(x)}{a} + C$$

24. $$\int \frac{f'(x)}{a^2 - [f(x)]^2}dx = \frac{1}{2a}1n\left|\frac{a+f(x)}{a-f(x)}\right| + C = \frac{1}{a}\tanh^{-1}\frac{f(x)}{a} + C$$

25. $$\int \frac{f'(x)}{[f(x)]^2 + a^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + C$$

26. $\int \frac{f'(x)}{\sqrt{a^2 - [f(x)]^2}} dx = \sin^{-1} \frac{f(x)}{a} + C$

27. $\int \frac{f'(x)}{\sqrt{[f(x)]^2 + a^2}} dx = \lambda n \left| f(x) + \sqrt{[f(x)]^2 + a^2} \right| + C = \sinh^{-1} \frac{f(x)}{a} + C$

28. $\int \frac{f'(x)}{\sqrt{[f(x)]^2 - a^2}} dx = \lambda n \left| f(x) + \sqrt{[f(x)]^2 - a^2} \right| + C = \cosh^{-1} \frac{f(x)}{a} + C$

29. $\int f'(x)\sqrt{a^2 - [f(x)]^2}\, dx = \frac{a^2}{2} \sin^{-1} \frac{f(x)}{a} + \frac{f(x)}{2} \sqrt{a^2 - [f(x)]^2} + C$

30. $\int f'(x)\sqrt{[f(x)]^2 + a^2}\, dx = \frac{a^2}{2} \lambda n \left| f(x) + \sqrt{[f(x)]^2 + a^2} \right| + \frac{f(x)}{2} \sqrt{[f(x)]^2 + a^2} + C$

31. $\int f'(x)\sqrt{[f(x)]^2 - a^2}\, dx = -\frac{a^2}{2} \lambda n \left| f(x) + \sqrt{[f(x)]^2 - a^2} \right| + \frac{f(x)}{2} \sqrt{[f(x)]^2 - a^2} + C$

32. **Integration by parts**

$\int u dv = uv - \int v du$

Priorities for *u*: inverse functions; log functions; x^n; e^{kx}; others

33. **Fundamental theorem of integration**

$\int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a)$

34. Mean value $= \frac{1}{b-a} \int_a^b f(x)\, dx$

35. $(\text{R.M.S})^2 = \frac{1}{b-a} \int_a^b (f(x))^2 dx$

J. INTEGRATION APPLICATIONS

VOLUMES OF SOLIDS OF REVOLUTION

For a vertical strip selection:

1. Disc: $V = \pi \int_a^b y^2 dx$
2. Washer: $V = \pi \int_a^b (y_2{}^2 - y_1{}^2) dx$
3. Shell: $V = 2\pi \int_a^b xy dx$