UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS
BACCALAUREUS TECHNOLOGIA
ENGINEERING: Electrical

MODULE MAT1AW4
ENGINEERING MATHEMATICS 4

CAMPUS **DFC**

JULY EXAMINATION 2018

DATE: JULY 2018	**SESSION: 8:30 – 11:30**
ASSESSOR:	**MRS H. Kotze**
MODERATOR:	**DR. E. Voges**
DURATION: 3 hours	**MARKS: 100**

NUMBER OF PAGES: **3 PAGES**

INSTRUCTIONS: **Calculators are allowed**
All answers must be completed in exam script

REQUIREMENTS: **Mathematics information booklet**
Answer scripts

SECTION A: Z-TRANSFORMATIONS

QUESTION 1 Determine the following z-transforms:

1.1 If $x(k)=\begin{cases}1 & k>1\\ 0 & k\le 1\end{cases}$;

1.1.1 Find $z\{x(k)\}$ (4)

1.1.2 Find $z\{x(k+1)\}$ (2)

1.2 If $x(k)=k-1$; find $z\{x(k-1)\}$ (3)

1.3 Find $z\left\{k\,e^{1-2k}\right\}$ (2)

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QUESTION 2 Determine the inverse z-transforms of the following using the method as indicated:

2.1 $x(k)=z^{-1}\left\{\dfrac{6z}{z^3+2z^2-5z-6}\right\}$

2.1.1 Use partial fractions to find $x(k)$. (8)

2.1.2 Now calculate $x(2)$. (1)

2.2 If $x(k)=z^{-1}\left\{\dfrac{2z^2}{(z+2)(z+1)^2}\right\}$

2.2.1 Use the residue method to find $x(k)$. (6)

2.2.2 Hence calculate $x(0)$. (1)

2.3 If $x(k)=z^{-1}\left\{\dfrac{3}{3+2z^{-1}-3z^{-3}}\right\}$, use the Power series method to find $x(0), x(1), x(2)$ and $x(3)$. (8)

[24]

QUESTION 3 Use z-transform methods to answer the following:

3.1 Given the difference equation $x(k+2)-2x(k+1)+x(k)=1$ with initial conditions $x(0)=0$ and $x(1)=\dfrac{3}{2}$.:

3.1.1 Find an expression for $x(k)$

3.1.2 Verify that $x(0)=0$ (9)

3.2 Given the backward difference equation $x(k)-x(k-1)=y(k)-2y(k-1)$ where $y(k)=\begin{cases}1 & k=0;\ k \text{ is even}\\ -1 & k \text{ is odd}\end{cases}$

3.2.1 Find an expression for $x(k)$

3.2.2 Now calculate $x(1)$.

(9)

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TOTAL SECTION A: 53

SECTION B PARTIAL DIFFERENTIAL EQUATIONS

QUESTION 4

Solve the partial differential equation

$$\frac{\partial^2 u(x,y)}{\partial x \, \partial y} = \frac{\partial u(x,y)}{\partial x} + 2$$

subject to the Cauchy data $\dfrac{\partial u(x,0)}{\partial x} = x^2$ and $u(0,y) = 0$. Make use of the method of ***direct integration***. (15)

QUESTION 5

The one dimensional heat equation

$$\frac{1}{\pi^2}\frac{\partial u(x,t)}{\partial t} - \frac{\partial^2 u(x,t)}{\partial x^2} = 0$$

describes the temperature distribution $u(x,t)$ in a copper wire of length $\pi\,\mathrm{cm}$ where the initial temperature is

$$u(x,0) = \sin x \text{ for } 0 < x < \pi \text{ .}$$

If the two ends of the wire are insulated so that no heat flows from the ends it means the rate of change in temperature at $x = 0$ and $x = \pi$ is zero, that is

$$\frac{\partial u(0,t)}{\partial x} = \frac{\partial u(\pi,t)}{\partial x} = 0 \qquad \text{for } t \geq 0$$

Find an expression for the temperature distribution $u(x,t)$ in the copper wire by using the method of ***separation of the variables***. (23)

QUESTION 6

Solve $\dfrac{\partial^2 u}{\partial x^2} = \dfrac{1}{c^2}\dfrac{\partial^2 u}{\partial t^2}$ if $t > 0$ by using ***Laplace transforms*** with respect to t subject to the initial conditions $u(x,0) = 0$; $\dfrac{\partial u(x,0)}{\partial t} = x$ for $0 \leq x \leq 3$ and boundary conditions $u(0,t) = 0$ and $u(3,t) = 0$. (14)

TOTAL SECTION B: 52