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UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF APPLIED PHYSICS AND ENGINEERING MATHEMATICS

BACCALAUREUS OPTOMETRIAE

MODULE MAT01A1
MATHEMATICS 1A

CAMPUS **DFC**

JULY SUPPLEMENTARY EXAMINATION

DATE 20/07/2018 **SESSION 08:00 – 10:00**

ASSESSOR **MR IK LETLHAGE**

INTERNAL MODERATOR **DR SM SIMELANE**

DURATION 2 HOURS **MARKS 70**

SURNAME AND INITIALS: ______________________

STUDENT NUMBER: ______________________

CONTACT NUMBER: ______________________

NUMBER OF PAGES: 15 (VERIFY THAT THE NUMBER OF PAGES IN YOUR SCRIPT IS CORRECT)

REQUIREMENTS : NON-PROGRAMMABLE SCIENTIFIC CALCULATOR

INSTRUCTIONS : ANSWER ALL THE QUESTIONS
USE THE BLANK PAGES AT THE BACK TO DO ROUGH WORK
NO PAGES SHOULD BE REMOVED FROM THIS PAPER.
USE ONLY BLUE OR BLACK INK TO WRITE AND DRAW. NO PENCIL.

QUESTION 1 [3]

Solve the inequality $\frac{2x-3}{x+2}<1$ and give the solution in interval form.

QUESTION 2 [6]

(a) Prove the trigonometric identity $\sin x \sin 2x + \cos x \cos 2x = \cos x$. (3)

(b) If $\sin x = \frac{1}{3}$ and $\sec y = \frac{5}{4}$, where x and y lie between 0 and $\frac{\pi}{2}$, evaluate the expression $\cos(x - y)$. (3)

QUESTION 3 [5]

(a) Evaluate the telescoping sum $\sum_{i=1}^{30}\left[(1+i)^2 - i^2\right]$. (2)

(b) Find the value of $\sum_{i=1}^{n}(3+2i)^2$. (3)

QUESTION 4 [6]

(a) Consider two complex numbers $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$.

Show that $z_1 z_2 = r_1 r_2\left[\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)\right]$ (3)

(b) Find all solutions of the equation $z^2 + z + 2 = 0$, z is a complex number. (3)

QUESTION 5 [5]

(a) Consider the following argument

If $x > 2$, then $x^2 > 4$

$x \leq 2$

$\therefore x^2 \leq 4$

Determine if this argument is valid. If it is valid, state the inference rule applied. If it is not valid, state whether the converse error or inverse error was committed. (2)

(b) Construct a truth table to determine if the propositional formula given is a tautology or a contradiction or neither: $\neg p \wedge \neg(p \rightarrow q)$ (3)

p	q	
T	T	
T	F	
F	T	
F	F	

QUESTION 6 [2]

(a) Translate the statement into natural language (English). (1)

"$\forall x \in \mathrm{R}\left(x > 0 \rightarrow x^2 > x\right)$" . R denotes the set of all real numbers.

(b) Write the negation of the following statement in predicate language. (1)

"Every integer is divisible by a prime number."

QUESTION 7 [3]

Use **proof by contradiction** to prove this statement:

If a is a rational number and b is an irrational number, then $a+b$ is an irrational number. (3)

QUESTION 8 [4]

(a) Let $f(x) = 3x + \ln x$. Find $f^{-1}(3)$. (2)

(b) If $f(x) = x^2 - 18x + 80$ and $g(x) = \sqrt{x+2}$, find $g \circ f$. (2)

QUESTION 9 [3]

Evaluate the following limit, **without** using l'Hôspital's Rule:

$$\lim_{t \to 0} \left[\frac{1}{t\sqrt{1+t}} - \frac{1}{t} \right].$$

QUESTION 10 [4]

Prove: If a function f is differentiable at $x = a$ then f is continuous at $x = a$.

QUESTION 11 [5]

Let $f(x) = 5x - x^2$.

(a) Use the definition of the derivative of a function to calculate $f'(2)$. (3)

(b) Find the equation of the line that is perpendicular to the tangent line to $y = 5x - x^2$ at $(2;6)$, that passes through the point of tangency. (2)

QUESTION 12 [3]

Find the derivative of $f(x) = \dfrac{e^{\cos 9x}}{\sqrt{x^2+1}}$.

QUESTION 13 [4]

Find $\frac{dy}{dx}$ if $y = (\ln x)^{\sinh x}$.

QUESTION 14 [3]

Use implicit differentiation to find $\frac{dy}{dx}$ if $x^2 + 3xy + y^2 = 6$.

QUESTION 15 [3]

Prove that $\frac{d}{dx}\left(\cosh^{-1} x\right) = \frac{1}{\sqrt{x^2 - 1}}, \quad x > 1.$

QUESTION 16 [3]

Given $f'(x) = 4 - \frac{3}{x^2 + 1}, \ f(1) = 0,$ find $f(x)$.

QUESTION 17 [2]

Use the Fundamental Theorem of Calculus Part 1, to evaluate $\frac{d}{dx}\int_1^{e^x} \ln t\,dt$.

QUESTION 18 [8]

Evaluate the following integrals.

(a) $\int \frac{2x^7 - 5x^4 + 2}{x^7}dx$ (3)

(b) $\int_{e}^{e^2} \frac{\ln x}{x} dx$ (3)

(c) $\int x\sqrt{9-x^2}\,dx$ (2)

TOTAL AVAILABLE MARKS: 72

USE THIS SPACE TO RE-DO ANY QUESTION YOU MAY HAVE CANCELLED