UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF PURE AND APPLIED MATHEMATICS

MODULE: MAT1D01/ MAT1DB1
ADVANCED BIO & ENVIRO MATHEMATICS & STATISTICS

CAMPUS: APK

ASSESSMENT: FINAL SUMMATIVE ASSESSMENT

DATE: 20 NOVEMBER 2017

ASSESSOR: MR M SIAS

INTERNAL MODERATOR: MR V VAN APPEL

DURATION: 2 HOURS **MARKS : 65**

SURNAME AND INITIALS_____

STUDENT NUMBER_____

CONTACT NUMBER_____

NUMBER OF PAGES: 12

INSTRUCTIONS:

- **ANSWER ALL THE QUESTIONS ON THE PAPER IN PEN.**
- **CALCULATORS ARE PERMITTED.**
- **ROUND OFF ALL ANSWERS TO TWO DECIMAL PLACES, UNLESS OTHERWISE SPECIFIED.**
- **SHOW ALL THE NECCESARY CALCULATIONS CLEARLY.**

1. A bacterial population that obeys the discrete – time dynamical system

$$b_{t+1} = rb_t$$

with the initial condition b_0 has the solution $b_t = b_0 r^t$. For the following values of r and b_0, state which population increases to infinity and which decreases to 0. For the population increasing to infinity, find the time when it will reach 10^{10}. For the population decreasing to 0, find the time when it will reach 10^5:

[6]

1.1 $b_0 = 10^9$ and $r = 1.2$. (3)

1.2 $b_0 = 10^{10}$ and $r = 0.7$ (3)

2. Find the leading behaviour of the following function at 0 and ∞: [2]

$$h(b) = \frac{119}{b^{15}} + \frac{43}{b^{13}} + 4e^{-2b} + 11e^{-5b}$$

3. Use L'Hopital's rule to evaluate the following limit: [5]

$$\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^{x^2}$$

4. Evaluate the following indefinite integral:

$$\int \frac{x}{(x-4)(-2x+3)} dx,$$

by firstly decomposing the integrand into partial fractions. [5]

5. Use integration by parts to evaluate the following definite integral: [5]

$$\int_1^2 x \ln x dx$$

(Hint: $\int \ln x dx = x \ln x - x + c$)

6. Find the area under the curve $H(y) = (2+5y)^{-2}$ from $y = 0$ to $y = 2$. [5]

7. Consider a population $P = P(t)$ with constant birth and death rates α and β, respectively, and a constant migration rate m, where α, β, and m are positive constants. Assume that $\alpha > \beta$. Then the rate of change of the population at time t is given by

$$\frac{dP}{dt} = kP - m, \qquad \text{where } k = \alpha - \beta \qquad [5]$$

7.1 Find the solution of this differential equation that satisfies the initial condition $P(0) = P_0$. (3)

7.2 What conditions on m will lead to an exponential expansion of the population? (2)

8. Suppose that the annual number of earthquakes registering 2.5 or more on the Richter scale and having an epicentre within 65 kilometres of a city follows a Poisson distribution with an average rate of $\lambda = 6.5$ per annum. Use the Central Limit Theorem to approximate the probability that there will be nine or more such earthquakes in the coming year. (Hint: for a Poisson random variable, X, $E(X) = \lambda$ and $Var(X) = \lambda$). [4]

9. Suppose that $\hat{\mu} = 41.5$ is the sample mean of 23 measurements from a normal distribution with given standard deviation $\sigma = 17$. [5]

   9.1. Determine the 95% confidence interval for the true mean. (2)

9.2. Determine the 99% confidence interval for the true mean. (2)

9.3. Provide an explanation for the difference in 9.1 and 9.2. above. (1)

10. Previous research has shown that the average lifespan of a zebra is 25 years with a sample standard deviation of 2.1 years. The average lifespan of a sample of 25 zebra is found to be 24 years. [13]

10.1 State the hypotheses to test whether the average lifespan of a zebra is less than 25 years (2)

10.2 State the formula of the test statistic to be used. Motivate your answer. (2)

10.3 Draw the rejection region and state the rejection rule for the null hypothesis for a 5% level of significance. (3)

10.4 Calculate the value of the test statistic. (1)

10.5 State your conclusion, motivate and interpret your findings. (3)

10.6 What type of error are we subject to? Describe, with reference to this specific question, when we would commit this error. (2)

11. The annual global concentrations for Carbon Dioxide were recorded as follows: [6]

	YEAR	CARBON DIOXIDE CONCENTRATION (ppm)
1	1950	310
2	1960	320
3	1970	325
4	1980	348
5	1990	350
6	2000	375
7	2010	385

11.1 Calculate and interpret the correlation coefficient. (2)

11.2 Determine the least square regression line to predict the Carbon Dioxide concentration on an annual basis. (2)

11.3 Calculate what the Carbon Dioxide concentration was for 1987. State whether this result is reliable or not. Explain your answer. (2)

12. A manufacturer of nicotine products to aid smokers to stop smoking believes that 40% of his customers prefer nicotine gum, 50% prefer nicotine patches and the remainder prefer nicotine inhalers. After conducting a survey, he found that 250 participants prefer the gum, 320 the patches and 120 the inhalers. Calculate the value of the sample test statistic. [4]