FACULTY OF SCIENCE

DEPARTMENT OF PURE AND APPLIED MATHEMATICS

MODULE	MAT1CA1 BIO & ENVIRO MATHEMATICS AND STATISTICS
CAMPUS	APK
ASSESSMENT	2017 JULY SUPPLEMENTARY EXAMINATION

ASSESSOR(S)	MR. S. MAFUNDA
MODERATOR(S)	MR. V. VAN APPEL
DURATION 2 HOURS	MARKS 75

SURNAME AND INITIALS ______________________________

STUDENT NUMBER ______________________________

CONTACT NUMBER ______________________________

NUMBER OF PAGES: 1 + 16 PAGES + FORMULAE AND STATISTICS TABLES

INSTRUCTIONS:
1. ANSWER ALL THE QUESTIONS ON THE PAPER IN PEN.
2. CALCULATORS ARE ALLOWED.
3. SHOW ALL CALCULATIONS AND MOTIVATE ALL ANSWERS.
4. FORMULA AND STATISTICS TABLES SHEETS ATTACHED.
5. IF YOU REQUIRE EXTRA SPACE, CONTINUE ON THE ADJACENT BLANK PAGE AND INDICATE THIS CLEARLY.

Question 1 [10 marks]

For questions 1.1 – 1.8, choose **one** correct answer, and make a cross (X) in the correct block.

Question	**a**	**b**	**c**	**d**	**e**
1.1					
1.2					
1.3					
1.4					
1.5					
1.6					
1.7					
1.8					

1.1 The domain of the function $f(x) = \frac{\sqrt{4-x^2}}{2-x}$ is: (1)

(a) $[-2, 2]$
(b) $(-\infty, 2) \cup (2, \infty)$
(c) $(-2, 2)$
(d) $[-2, 2)$
(e) None of the above

1.2 The following infinite limit is equal to:

$$\lim_{x\to-\infty} \frac{x^4 - x^3 - 8x}{7x^2 + 10}$$

(1)

(a) $-\frac{1}{7}$
(b) 0
(c) ∞
(d) $\frac{1}{2}$
(e) None of the above

1.3 If $f(x) = 5e^{4x} + (2x-1)^e$ then, $f'(x) = \ldots$ (1)

(a) $5e^{4x} + (2x-1)^e$
(b) $20e^{4x} + e(2x-1)^{e-1}$
(c) $20e^{4x} + 1$
(d) $20e^{4x} + 2e(2x-1)^{e-1}$
(e) None of the above

1.4 If the function $y = \frac{4}{\sqrt{8x+6}}$ is a composite function, $(fog)(x)$, then: (2)

(a) $f(x) = \sqrt{8x+6}, \quad g(x) = 4$
(b) $f(x) = 4x, \quad g(x) = \sqrt{8x+6}$
(c) $f(x) = \frac{4}{x}, \quad g(x) = 8x+6$
(d) $f(x) = \frac{4}{\sqrt{x}}, \quad g(x) = 8x+6$
(e) None of the above

1.5 For any random variable X with finite expectation and variance. The $Cov(X, X)$ is... (1)

(a) equal to $E(X)$
(b) equal to $Var(X)$
(c) equal to 0
(d) does not exist
(e) None of the above

1.6 Let $S = \{0, 1, 3, 5, 6, 7\}$ be the sample space of an experiment and let $A = \{0, 3, 5, 7\}$, $B = \{0, 3, 5, 6, 7\}$, $C = \{0, 5, 6\}$, $D = \{1\}$ be the events in this experiment, then... (1)

(a) A and B are mutually exclusive
(b) $\bar{A} \cap B = \emptyset$
(c) $A \cap B = \{0, 5\}$
(d) B and D are mutually exclusive
(e) None of the above

1.7 A Poisson Distribution (1)

(a) has $E(X) = Std.Dev.(X)$
(b) $X \sim Po(1)$ then $P(X \geq 5) = 0.0037$
(c) $X \sim Po(1)$ then $P(X \geq 5)$ does not exist
(d) describes a continuous random variable
(e) None of the above

1.8 There are two telephone lines- A and B. Line A is engaged 50% of the time and line B is engaged 60% of the time. Both lines are engaged 30% of the time. The probability that line A is engaged, but line B is not engaged is: (2)

(a) 0.2

(b) 0.5

(c) 0.4

(d) 0.1

(e) None of the above

Question 2 [4 marks]

According to the *Guinness Book of Records, 1990*; The men's Olympic record for the 1500 meters was 216.8 seconds in 1972 and 215.9 seconds in 1988.

2.1 Find the equation $f(x)$ of the straight line connecting these events. (1.5)

2.2 Compute the inverse of the above equation. (2)

2.3 State whether or not the inverse is a function. (0.5)

Question 3 [3 marks]

Solve for x:

$$\frac{1025}{8 + \ln(4x)} = 5$$

(3)

Question 4 [4 marks]

For what value of the constant c is the function f continuous on $(-\infty, \infty)$?

$$f(x) = \begin{cases} x^3 + c & \text{if } x \leq 3 \\ cx - 5 & \text{if } x > 3 \end{cases}$$

(4)

Question 5 [3 marks]

Use the definition of derivatives (first principle) to find $f'(x)$, given:

$$f(x) = \sqrt{x} \text{ where } x > 0$$

(3)

Question 6 [5 marks]

Evaluate the following:

6.1 $f'(x)$ if $f(x) = 3x\sin(2x)$ (2)

6.2 $f''(x)$ if $f(x) = e^{3x^3+2x^2+x} + 5\ln(3x)$ (3)

Question 7 [4 marks]

The concentration of a certain drug in the bloodstream t minutes after swallowing a pill containing the drug can be approximated using the equation

$$C(t) = \frac{1}{9}(3t+1)^{-\frac{1}{2}}$$

where $C(t)$ is the concentration in arbitrary units and t is in minutes. Find the rate of change of concentration with respect to time at $t = 5$ minutes, in units per minute. (4)

Question 8 [11 marks]

100 pairs of plants are crossed, and each pair produces ten offsprings. The number of tall offspring is then counted. Now suppose we gather the following information from this experiment:

Number of Tall Offspring	Data
0	5
1	11
2	35
3	27
4	13
5	8
6	1
7	0
8	0
9	0
10	0

8.1 Find and draw a histogram of the **probability** of each simple event. (2)

8.2 Which simple event(s) is most likely? (0.5)

8.3 Find and graph the cumulative distribution. (2)

8.4 Find the expectation of the data. (*Hint: Use the probabilities in (8.1) and* $\sum_{x \in X} xP(x)$). (2)

8.5 Use (8.4) to find the variance and standard deviation of the data. (2.5)

8.6 Find the meadian and mode of the data. (2)

Question 9 [6 marks]

9.1 State, without proof, the Law of Total Probability. (2)

9.2 Careful onservation reveals that your probability of receiving a phone call during any 20-min interval whileyou are awake is 0.1. You spend 20 minutes per day in the shower (which is 2% of your working hours) and during which time the phone rings with probability 0.2.

9.2.1 Are the events of showering and receiving phone calls independent? Prove it (SHOW ALL WORKINGS). (1.5)

9.2.2 Find the probability of receiving a phone call during a 20-min period when you are not in the shower. (2.5)

Question 10 [7 marks]

A new surgical procedure is said to be successful 80% of the time. Suppose the operation is performed sixteen times. And the results are assumed to be independent of one another, find the following probabilies.

10.1 Exactly 2 operations are successful, i.e, $P(X = 2)$. (2)

10.2 Using the cumulative tables, find the probability that at most three of these operations are successful. (3)

10.3 Using the cumulative tables, find the probability that between two and thirteen operations are successful. (2)

Question 11 [7 marks]

The number of people entering the intensive care unit at a particular hospital on any one day has a poisson probability distribution with a mean equal to five people per day. Find the probability that on a given day...

11.1 Exactly two people enter the intensive care unit, i.e $P(X = 2)$ (2)

11.2 Using the cumulative tables find the probability that at most three people enter. (3)

11.3 Using the cumulative tables, the probability that between two and thirteen people enter.(2)

Question 12 [7 marks]

12.1 The stem diameters at base of a particular species of sunflower plant have a normal distribution with average diameter of 35 millimeters (mm) and a standard deviation of 3 mm.

12.1.1 If a sunflower plant is selected at random, calculate the probability that the stem will measure more than 42 mm in diameter. i.e $P(X > 42)$. (2)

12.1.2 If a sunflower plant is selected at random, calculate the probability that the stem will measure between 30 and 34 mm in diameter. (2)

12.2 If 5% of sunflower plant described in 12.1 with the thinnest stems are used as compost, determine the maximum diameter of a stem of a sunflower plant that would be used as compost. (3)

Question 13 [4 marks]

Consider the following data for cell age A and the number of toxic molecules N inside.

	A=0	A=1	A=2	A=3	A=4
N=0	0.03	0.00	0.09	0.06	0.09
N=1	0.01	0.05	0.06	0.12	0.15
N=2	0.02	0.03	0.06	0.08	0.15

Find the correlation of A with N. *SHOW* ***ALL*** *WORKINGS.* (4)

Bonus Question [5 marks]

When we consider the flow of blood through a blood vessel, such as a vein or artery, we can model the shape of the blood vessel by a cylindrical tube with radius R and length l. Because of friction at the walls of the tube, the velocity of the blood is greatest along the central axis of the tube and decreases as the distance from the axis increases until becomes 0 at the wall. There is a relationship between ν and r, and is given by the law of laminar ow. This law states that

$$\nu = \frac{P}{4\eta l}(R^2 - r^2)$$

where η is the viscosity of the blood and P is the pressure difference between the ends of the tube. If P and l are constant, then ν is a function of r with domain $[0, R]$.
Consider a blood vessel with radius, $R = 0.01$ cm, length $l = 3$ cm, pressure difference $P = 3000$ dynes/cm^2, and viscosity $\eta = 0.027$.

(a) Find the velocity of the blood at the wall of the tube. ($i.e\ at\ r = R = 0.01$). (1.5)

(b) Find the instantaneous rate of change of the velocity with respect at the wall of the tube. ($i.e\ at\ r = R = 0.01$). (3.5)