UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF PURE AND APPLIED MATHEMATICS

MODULE **MAT2EC1 / MAT1C2E** (CALCULUS SECTION)
BIO & ENVIRO MATHS AND STATS

CAMPUS **APK**

SUPPLEMENTARY EXAM **JANUARY 2018**

DATE: JANUARY 2017 **SESSION:**

ASSESSOR: MR. T. MOHUBEDU

INTERNAL MODERATOR: MR. V. VAN APPEL

DURATION: 60 MINUTES **MARKS:** 50

SURNAME AND INITIALS: ______________________________

STUDENT NUMBER: ______________________________

CONTACT NUMBER: ______________________________

Please read the following instructions carefully

1. Answer all questions on the paper in pen.
2. This paper consists of 11 pages including the cover page.
3. **Show all calculations.**
4. **Calculators are allowed.**

1 Convert $15°$ to radian measure. [1]

2 Given $f(x) = x^2 - 3$ and $g(x) = 2x$

2.1 Find $f(a + 1)$ [1]

2.2 Find the product: $g\ .\ f$ [2]

3 Solve for x: $3\ e^{1.2.x} = 1$ [2]

4. Find the equation of a straight line that is passing through the points $(-2, 1)$ and $(0,\ 3)$. [2]

5 A population follows the discrete – time dynamical system $b_{t+1} = rb_t$ with $r = 0.75$ and $b_0 = 5.0 \times 10^6$.

5.1 Find the solution of the system and express it in exponential form. [1]

5.2 When will the population reach 1.5×10^3? [2]

6 The number of mosquitos (M) that ends up in a room is a function of how far the window is open $(W, in\ cm^2)$ according to $M(W) = 2 + 7W$. The number of bites (B) depends on the number of mosquitos according to $B(M) = 0.6M$

6.1 Find the formula for the number of bites as a function of how far the window is open. [2]

6.2 How many bites will you get if the window closed? [1]

6.3 How many bites will you get if the window was 8 cm^2 open? [1]

6.4 Find the formula of how far the window is open, as a function of the number of mosquitos in the room. [2]

7 A population has a half – life of 5 years and an initial size of $50\ 000$.

7.1 What is the population in 10 years? [1]

7.2 Find the equation for population size $P(t)$ as a function of time. [2]

7.3 Find the average rate of change in population size during the fifth year. [2]

9 Set up a table to estimate the limit: $\lim_{x \to 0} \dfrac{x^2}{\cos x - 1}$ [2]

10 Given $f(x) = x^2 - 3$

10.1 Find $f'(x)$ [1]

10.2 Find the critical values of f. [1]

10.3 Find the interval of increase and decrease. [1]

11. Suppose a population $V(t)$ of viruses ($in\ millions$) in an infected person is dying according to $V(t) = 2.0e^{0.4t}$ where time t is measured in hours.

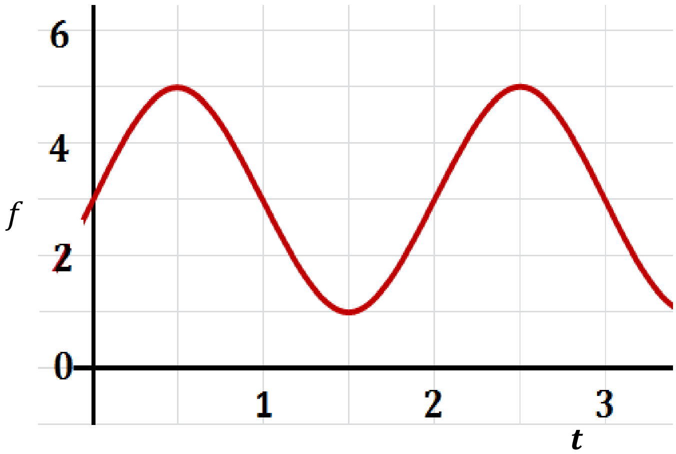
11.1 Sketch the graph of the population $V(t)$ for $0 \leq t \leq 4$ [3]

11.2 Calculate the time at which the number of viruses will reach 8.0 million. [2]

12 Simplify $\log_4(3 \cdot 2^3) - \log_4 24$ (using laws of logarithm) [2]

13 Consider the given sinusoidal graph of f, find the following: [4]

13.1 the average,



13.2 amplitude,

13.3 period

13.4 phase.

13.2 maximum,

13.6 minimum,

13.7 the equation of V.

14 Given $f(t) = 2[1 + 0.5\cos(\pi t - \pi)]$

14.1 Write f in standard form [1]

14.2 Sketch the graph of f for $0 \leq t \leq 4$ [3]

15 Sleepiness has two cycles, a circadian rhythm (S_c) with a period of approximately 24 hours and an ultradian rhythm (S_u) with a period of approximately 4 hours. Both have phase 0 (starting at midnight) and average 0, but the amplitude of circadian rhythm is 1.0 sleepiness unit and the ultradian is 0.5 sleepiness unit.

15.1 Write down the equation of circadian rhythm [1]

15.1 Write down the equation of ultradian rhythm [1]

15.3 Sketch the graph of the two cycles combined, for $0 \leq t \leq 12$. [4]

15.4 At what time of the day are you sleepiest? [1]

15.5 At what time of the day are you least sleepy? [1]

---------[TOTAL 50]