



**FACULTY OF SCIENCE
FAKULTEIT NATUURWETENSKAPPE**

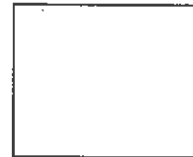
DEPARTMENT OF PURE AND APPLIED MATHEMATICS

MODULE MAT1A2E

(Calculus of One-Variable Functions I)

CAMPUS APK

EXAM SUPPLEMENTARY EXAMINATION – JANUARY 2018



DATE: 08 JANUARY 2018

SESSION: 08:30 – 12:30

ASSESSOR:

**MS. D. SCHUBERT
MS. T. OBERHOLZER**

MODERATOR:

MR. J. HOMANN

DURATION: 2 HOURS

MARKS: 50

SURNAME AND INITIALS _____

STUDENT NUMBER _____

CONTACT NUMBER _____

NUMBER OF PAGES: 13 PAGES (Including front page)

**INSTRUCTIONS: ANSWER ALL THE QUESTIONS IN PEN ON THE EXAM PAPER.
CALCULATORS ARE NOT ALLOWED.**

Question 1

Find the domain of $f(x) = \sqrt{\frac{x^2-3x-4}{x^2+1}}$. (3)

Question 2

Find $\lim_{k \rightarrow 0} \frac{\sin 3k}{\sin 7k}$ without using L'Hospital's Rule. (2)

Question 3

Let
$$g(x) = \begin{cases} B^3x^2; & x < 1 \\ B^2 - 2Bx; & x \geq 1 \end{cases}$$

Find B given that g is continuous at 1.

(3)

Question 4

(a) Find $f'(2)$ from First Principles if $f(x) = \frac{2}{\sqrt{3x-1}}$. (3)

(b) Hence, find the equation of the tangent to f at $x = 2$. (2)

Question 5

Find the point(s) where the tangent(s) to $h(d) = \frac{2d}{d^4+3}$ is / are horizontal. (3)

Question 6

Differentiate:

(a) $k(x) = \frac{e^{x^2}}{(3x-4)^5}$ (2)

(b) $xy + ex = -\ln(xe^{-2x}) + \sin y$ (3)

Question 7

Evaluate:

(a) $\lim_{x \rightarrow -\frac{\pi}{2}} (-\tan x)$ (1)

(b) $\lim_{p \rightarrow -\infty} \frac{\sqrt{p^2+3}}{p^2-1}$ (2)

(c) $\lim_{x \rightarrow \infty} \left(\frac{\sin 3x}{x^2} \right)$ (2)

Question 8

Find the inverse of $f(x) = 3 \ln(2x - 5) - 1$

(2)

Question 9

Determine: $\cot(\sec^{-1} \frac{8}{3})$

(2)

Question 10

Prove the Quotient Rule.

(4)

Question 11

Simplify: $\neg[\forall x(x > 0 \vee x = x^4)]$

(2)

Question 12

Consider the following statement: "No swimmers drink alcohol."

(a) Translate the statement into formal language.

(1)

(b) Write an informal negation of the statement.

(2)

Question 13

Are the following statements TRUE or FALSE? Substantiate your answer.

(a) $\exists k \in \mathbb{Z}$ such that $\sqrt[4]{k} = k$ (1)

(b) $\forall j \in \mathbb{R}, j^6 \geq j$ (1)

Question 14

Compute the following:

(a) The first 3 terms of $a_n = \frac{2-(-1)^{2n}}{n^2}; n \geq 0$. (2)

(b) $\sum_{i=3}^5 (\frac{1}{2-i} - \frac{1}{i})$. (2)

Question 15

(a) Prove that if a is rational and b is irrational, then $(a + b)$ is irrational. (2)

(c) Prove that $\sum_{i=1}^n i^3 = \frac{n^2(n+1)^2}{4}$, for $n \geq 1$. (3)

END