UNIVERSITEIT
— VAN —
JOHANNESBURG

FACULTY OF SCIENCE
FAKULTEIT NATUURWETENSKAPPE

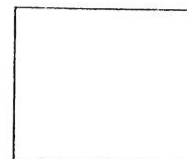## DEPARTMENT OF PURE AND APPLIED MATHEMATICS

MODULE   MAT1A2E

              (Calculus of One-Variable Functions I)

CAMPUS   APK

EXAM   EXAMINATION - NOVEMBER 2017

DATE: 11 November 2017   SESSION: 12:30 – 14:30

ASSESSOR: MS. D. SCHUBERT
                                           MS. T. OBERHOLZER

MODERATOR: MR. J. HOMANN

DURATION: 2 HOURS   MARKS: 50

SURNAME AND INITIALS _____

STUDENT NUMBER _____

CONTACT NUMBER _____

NUMBER OF PAGES: 12 PAGES   (Including front page)

INSTRUCTIONS: ANSWER ALL THE QUESTIONS IN PEN ON THE EXAM PAPER.
                            CALCULATORS ARE NOT ALLOWED.

Question 1

Is $h(a) = \sqrt[3]{\dfrac{\cosec a}{a^7}}$ even, odd or neither? (2)

Question 2

Find the domain of $f(x) = \sqrt[4]{\dfrac{x^2-4x-5}{x^3+1}}$. (4)

Question 3

Prove $\quad \lim\limits_{x \to 0} \dfrac{sinx}{x} = 1$, without using L'Hospital's Rule. $\hspace{2cm}$ (4)

Question 4

Find $\lim\limits_{k \to 0} \dfrac{sin3k}{sin5k}$, without using L' Hospital's Rule. (2)

Question 5

Sketch: $f(x) = \begin{cases} -2x & if & x \leq -2 \\ 2^{-x} & if & -2 < x \leq 3 \\ \sqrt{x+3} & if & x > 3 \end{cases}$ (3)

Question 6

Let $g(x) = \begin{cases} B^3 x^2; & x < 1 \\ B^2 - 2Bx; & x \geq 1 \end{cases}$

Find B given that g is continuous at 1. (3)

Question 7

(a)   Find $f'(2)$ from First Principles if $f(x) = \dfrac{3}{4x-1}$.   (3)

(b)   **Hence,** find the equation of the tangent to f at $x = 2$.   (2)

Question 8

Find the point(s) where the tangent(s) to

$h(d) = \dfrac{2d}{d^1+3}$ is /are horizontal. (3)

Question 9

Differentiate:

(a) $y = \log_5(e^{3.x})$ (2)

(b) $y = 2\sin^{-1}(\sqrt{5x})$ (2)

(c) $y = \dfrac{x^3 \ln 2x}{\sin x}$, using logarithmic differentiation.   (4)

Question 10

Determine:

(a) $\displaystyle\lim_{x \to -\infty} \frac{\sqrt{1+x^8}}{x^4+1}$ (2)

(b) $\displaystyle\lim_{u \to 2} \frac{\sqrt{4u+1}-3}{u-2}$ (3)

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(ii) Prove that, for $n > 1$,

$$1 \times 2 + 2 \times 3 + 3 \times 4 + \ldots + n(n+1) = \frac{n(n+1)(n+2)}{3}$$

(5)

(iii) Determine whether the following argument is valid? [Use a Truth table]

$p \wedge q \rightarrow \neg r$

$p \vee \neg q$

$\neg q \rightarrow p$

$\neg r$ (4)