UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF PURE AND APPLIED MATHEMATICS

MODULE	MAT01B1 APPLICATIONS OF CALCULUS
CAMPUS	APK
ASSESSMENT	SUPPLEMENTARY EXAM

DATE 12/01/2018	TIME 08:00
ASSESSOR(S)	DR A CRAIG MS S RICHARDSON
INTERNAL MODERATOR	MR S MAFUNDA
DURATION 2 HOURS	MARKS 70

SURNAME AND INITIALS ____

STUDENT NUMBER ____

CONTACT NUMBER ____

NUMBER OF PAGES: 1 + 12 PAGES

INSTRUCTIONS:
1. ANSWER ALL THE QUESTIONS ON THE PAPER IN PEN.
2. NO CALCULATORS ARE ALLOWED.
3. SHOW ALL CALCULATIONS AND MOTIVATE ALL ANSWERS.
4. IF YOU REQUIRE EXTRA SPACE, CONTINUE ON THE <u>ADJACENT</u> BLANK PAGE AND INDICATE THIS CLEARLY.

Question 1 [10 marks]

For questions 1.1 – 1.10, choose **one** correct answer, and make a cross (X) in the correct block.

Question	**a**	**b**	**c**	**d**	**e**
1.1					
1.2					
1.3					
1.4					
1.5					
1.6					
1.7					
1.8					
1.9					
1.10					

1.1 The Mean Value Theorem does not apply to $f(x) = |x - 3|$ on $[1, 4]$ because (1)

(a) $f(x)$ is not differentiable on $(1, 4)$

(b) $f(x)$ is not continuous on $[1, 4]$

(c) $f(1) \neq f(4)$

(d) $f(1) > f(4)$

(e) None of the above

1.2 Let f be a twice-differentiable function of x such that, when $x = c$, f is decreasing, concave up, and has an x-intercept. Which of the following is true? (1)

(a) $f(c) < f'(c) < f''(c)$

(b) $f'(c) < f''(c) < f(c)$

(c) $f''(c) < f(c) < f'(c)$

(d) $f'(c) < f(c) < f''(c)$

(e) None of the above

1.3 Which integral below gives the area of the surface obtained when the the curve $y = \ln x$ between $x = 1$ and $x = e$ is rotated about the y-axis? (1)

(a) $2\pi \int_1^e \frac{\ln x}{\sqrt{1 + x^2}} \, dx$

(b) $2\pi \int_1^e x\sqrt{1 + \frac{1}{x^2}} \, dx$

(c) $2\pi \int_1^e x\sqrt{1 + x^2} \, dx$

(d) $2\pi \int_1^e \ln x\sqrt{1 + x^2} \, dx$

(e) $2\pi \int_1^e x\sqrt{1 - \frac{1}{x^2}} \, dx$

1.4 The Mean Value Theorem for Integrals guarantees the existence of a special point on the graph of $y = 2x^3$ between $(0, 0)$ and $(2, 16)$. What are the co-ordinates of this point? (1)

(a) $(2^{\frac{2}{3}}, 4)$
(b) $(4, 2^{\frac{2}{3}})$
(c) $(2^{\frac{1}{3}}, 4)$
(d) $(4, 2^{\frac{3}{2}})$
(e) None of the above.

1.5 Which integral below gives the arc length of the curve $y = \tan x$ on the interval $[0, \frac{\pi}{4}]$? (1)

(a) $\displaystyle\int_0^{\frac{\pi}{4}} \sqrt{1 - \sec^4 x}\, dx$
(b) $\displaystyle\int_0^{\frac{\pi}{4}} \sqrt{1 + \sec^4 x}\, dx$
(c) $\displaystyle\int_0^{1} \sqrt{\frac{\pi}{4} + \sec^4 x\, dx}$
(d) $\displaystyle\int_0^{\frac{\pi}{4}} \sqrt{1 + \tan^2 x}\, dx$
(e) $\displaystyle\int_0^{\frac{\pi}{4}} \sqrt{1 + \sec^2 x \tan^2 x}\, dx$

1.6 The average value of $f(x) = \dfrac{1}{x}$ on $[1, e]$ is: (1)

(a) $\dfrac{1}{1-e} \ln|x|$
(b) $\dfrac{1}{e}$
(c) $\ln e - \ln 1$
(d) $\dfrac{1}{e-1}$
(e) None of the above.

1.7 Which function is a constant solution of the equation $\dfrac{dy}{dt} = y^4 - y^3 - 6y^2$? (1)

(a) $y(t) = 2$
(b) $y(t) = e^t$
(c) $y(t) = 3$
(d) $y(t) = 5$
(e) None of the above.

1.8 The function $f(x) = \dfrac{3x-6}{(x-1)(x^2-5x+6)}$ has the following asymptotes: (1)

(a) $x = 1,\ x = 2,\ x = 3$
(b) $x = 1,\ x = 2,\ y = 3$
(c) $x = 1,\ x = 3,\ y = 2$
(d) $x = 1,\ x = 3,\ y = 0$
(e) None of the above

1.9 The sum of two non-negative numbers is 9. What equations can be used to maximise the product of one number and the square of the other number. (1)

(a) $y = 9 - x,\ P = xy$
(b) $y = x - 9,\ P = xy^2$
(c) $y = x - 9,\ P = xy$
(d) $y = 9 - x,\ P = xy^2$
(e) None of the above

1.10 Find the arc length function for the function $f(x) = \cosh x$ with starting point $(0, 1)$. Select the correct answer. (1)

(a) $\sinh x$.
(b) $-\cosh x$.
(c) $-\sinh x\, dx$.
(d) $\cosh x$.
(e) None of the above.

Question 2 [3 marks]

Use the Binomial Theorem to expand $\left(\dfrac{1}{x} + 2x^2\right)^4$. Simplify all coefficients. (3)

Question 3 [10 marks]

(a) Evaluate $\displaystyle\int \frac{x^3}{\sqrt{x^2+25}}\,dx$ (4)

(b) Evaluate $\displaystyle\int \frac{3x-5}{x^2-2x-3}\,dx$ (3)

(c) Evaluate the integral $\displaystyle\int_0^1 \frac{x-1}{\sqrt{x}}\,dx$ or show that it diverges. (3)

Question 4 [3 marks]

Describe the concavity of the graph of $f(x) = x^2 + \dfrac{1}{x}$ and find the points of inflection, if any exist. (3)

Question 5 [3 marks]

Sketch the graph of a function that satisfies the following conditions:

(i) Domain $= (-\infty, 6)$

(ii) $f(0) = 0$, $f'(-2) = f'(1) = 0$

(iii) $\lim_{x\to 6^-} f(x) = -\infty$, $\lim_{x\to -\infty} f(x) = \infty$

(iv) $f'(x) < 0$ on $(-\infty, -2) \cup (1, 6)$ and $f'(x) > 0$ on $(-2, 1)$

(v) $f''(x) > 0$ on $(-\infty, 0)$ and $f''(x) < 0$ on $(0, 6)$

Question 6 [6 marks]

(a) State Rolle's Theorem. (2)

(b) The Mean Value Theorem states that if $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b) then there exists

$$c \in (a, b) \text{ such that } f'(c) = \frac{f(b) - f(a)}{b - a}.$$

Use Rolle's Theorem to prove the Mean Value Theorem. Include a sketch in your proof. (4)

Question 7 [6 marks]

(a) Calculate the area of the region bounded by the x-axis, $y = \cos x$ and $y = \sin 2x$ between $x = 0$ and $x = \dfrac{\pi}{2}$. Include a sketch of the region. (3)

(b) The region bounded by the curve $y^2 = 1 - x$ and the y-axis is rotated about the line $y = 2$. Use the method of **cylindrical shells** to calculate the volume of the solid generated. Include a sketch in your answer. (3)

Question 8 [7 marks]

(a) Solve the differential equation $y' = xe^{-\cos x} + y\sin x$. (4)

(b) Solve the initial value problem $\dfrac{du}{dt} = \dfrac{2t + \sec^2 t}{2u}, \quad u(0) = 4.$ (3)

Question 9 [4 marks]

The volume of a right circular cone of radius r and height h is $V = \frac{\pi}{3} r^2 h$.
Suppose that the radius and height of the cone are changing with respect to time t.

(a) Find a relationship between $\frac{dV}{dt}$, $\frac{dr}{dt}$, and $\frac{dh}{dt}$. (2)

(b) At a certain moment, the radius and height of the cone are 10 cm and 20 cm respectively. The radius and height are changing at rates of 0.2 cm/s and 0.3 cm/s respectively. At what rate is the volume of the cone changing? (2)

Question 10 [4 marks]

Consider the parametric curve $x = t^2 + 1$, $y = 4t^3 + 3$. Find the arc length of the curve for $0 \leqslant t \leqslant 1$. (4)

Question 11 [3 marks]

Write the following polar equation in Cartesian form and then describe its curve in words: (3)

$$r = 8\cos\theta + 6\sin\theta.$$

Question 12 [4 marks]

Consider the polar curve $r = \dfrac{1}{\theta}$. Find the slope of the tangent when $\theta = \pi$. (4)

Question 13 [7 marks]

(a) Find the focus of the parabola $y = -x^2 - 2x + 1$. (3)

(b) Find the centre, vertices and foci of the conic $4x^2 + y^2 - 8x + 4y - 8 = 0$. (4)