UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF PURE AND APPLIED MATHEMATICS

MODULE: MAFT0B2/MA2BFET

COURSE: MATHEMATICS 2B FOR TEACHERS

CAMPUS: APK

EXAM: NOVEMBER 2017

DATE: SATURDAY 11 NOVEMBER 2017

TIME: 08:30 – 10:30

ASSESSORS: MS. S. RICHARDSON
MS. T. OBERHOLZER

INTERNAL MODERATOR: MR. T. MOHUBEDU

DURATION: 2 HOURS **MARKS:** 100

SURNAME AND INITIALS

STUDENT NUMBER

CONTACT NUMBER

NUMBER OF PAGES: 14 PAGES (including front page)

INSTRUCTIONS: **ANSWER ALL THE QUESTIONS, CALCULATORS ARE NOT ALLOWED.**

Question 1: **[4 x 2 = 8]**

The following questions are multiple choice questions. There is only one correct answer from the choices given. Circle the correct option.

1.1 If

$$f(x) = \begin{cases} \sqrt{-x} & if \quad x < 0 \\ 3 - x & if \quad x \geq 0 \end{cases}$$

then f is discontinuous at $x = \cdots$

A. No value of x
B. $x = 0$
C. $x = 3$
D. x^2
E. None of these.

1.2 The slope of the tangent line to the graph of $\boldsymbol{y = (2 + x).\tan x}$ at the point $x = 0$ is:

A. 1
B. -1
C. 2
D. 0
E. None of these

1.3 If $f(x) = x^4 - 1, g(x) = \sqrt[4]{x^2 - 1}$ and $h(x) = \sqrt{x + 2},$ then $(f \circ g \circ h)(x) =$

A. x^2
B. $x\sqrt{x + 2}$
C. $2x$
D. x
E. None of these

1.4 If f is differentiable, then $\frac{d}{dx}\left[f(\sqrt{x})\right] =$

A. $\frac{f'(x)}{2\sqrt{x}}$
B. $f'(x).\sqrt{x}$
C. $\frac{f'(\sqrt{x})}{2\sqrt{x}}$
D. $\frac{f'(x)}{\frac{1}{2}\sqrt{x}}$
E. None of these

Question 2: **[4 x 2 = 8]**

Determine whether the following statements are true or false. **If it is false, explain why and give an example to illustrate the truth.**

Statement	True or False (& Explanation)
If functions f and g are such that $$f(x) = g(x) + k$$ where k is a constant, then $$f\,'(x) = g\,'(x) + k$$	
The derivative of $[g(x)]^2$ is equal to $[g\,'(x)]^2$.	
If $f(x)$ is a differentiable function then $$\lim_{x\to 4} \frac{f(x) - f(4)}{x - 4} = f'(4)$$	
If a function is discontinuous at $x = a$, then it has a vertical asymptote at $x = a$.	

Question 3: **[11]**

3.1 Determine the following limits (if they exist):

a.

$$\lim_{x \to -\infty} \frac{\sqrt{9x^8 - x + 1}}{3x^4 + 5x}$$

(3)

b.

$$\lim_{x \to 0} x \cot x$$

(2)

c.

$$\lim_{x \to -3} \left[\frac{x+1}{x+3}\right]$$

(3)

3.2 Use the Squeeze Theorem to determine the limit:

$$\lim_{x \to 0} x^2 \cos\frac{1}{x^2}$$

(3)

Question 4: **[7]**

4.1 Let

$$f(x) = \begin{cases} (x-1)^2 + 2 & if \quad |x| < 1 \\ 3x & if \quad x \geq 1 \end{cases}$$

a. Sketch the graph of f .

(3)

b. Determine

$$\lim_{x \to 1^-} f(x)$$

(1)

c. Determine

$$\lim_{x \to 1^+} f(x)$$

(1)

d. Determine

$$\lim_{x \to 1} f(x)$$

(1)

e. Is f continuous at $x = 1$? Motivate your answer.

(1)

Question 5: **[18]**

5.1 Use the limit definition of the derivative to show that:

$$\frac{d}{dx}(cosx) = -sinx$$

(5)

5.2 Use the rules of differentiation to find the first derivatives of the following functions (simplify your answers as far as possible):

a. Find $f'(x)$ if

$$f(x) = \sqrt{\frac{x}{x^2 + 1}}$$

(4)

b. Find $g'(x)$ if

$$g(x) \ = (sec\ x + tan\ x)^5$$

(2)

c. Find $f'(x)$ in terms of $g'(x)$ if

$$f(x) = x^8\ g(x).$$

(2)

5.3 Find all x-values, $x \in [0; 2\pi]$, at which the tangent line is horizontal on the graph of the function

$$y = 6\sin x + \sin^2 x.$$

(5)

Question 6: **[28]**

6.1 Calculate $\frac{dy}{dx}$, given

$$x \sin 2y - 3y \sec x = 1$$

(4)

6.2 A stone is dropped into a calm pool of water, causing ripples in the form of concentric circles. The radius r of the outer ripple is increasing at a rate of 1 cm/s. When the radius is 4 cm, at what rate is the total area A of the disturbed water changing?

(4)

6.3 The product of two positive numbers is 288. Minimise the **sum of the second number and twice the first number.**

(5)

6.4 The Nouken family wants to build a television room onto their house. The dad draws up the plans for the new square room of length k metres. The mum looks at the plans and decides that the area of the room needs to be doubled. To achieve this:

- Mum Rachel suggests doubling the length of the sides of the room
- Dad Taavi recommends adding 2 m to the length of the sides
- Daughter Ebele suggests multiplying the length of the sides by a factor of $\sqrt{2}$
- Son Boipelo suggests doubling only the width of the room

a. Make sketches to illustrate the original plan, and then each suggestion. Indicate clearly which sketch refers to which person's suggestion. Your sketches do not have to be to scale, but **clearly show the lengths of the sides for each.**

(5)

b. Calculate the area of each one of your sketches to determine whose suggestion will double the area of the square room. Show all calculations.

(5)

6.5 Find $f(x)$ if $f''(x) = -\cos x + 6$ and $f(0) = 3$ and $f'(\pi) = 6\pi$.

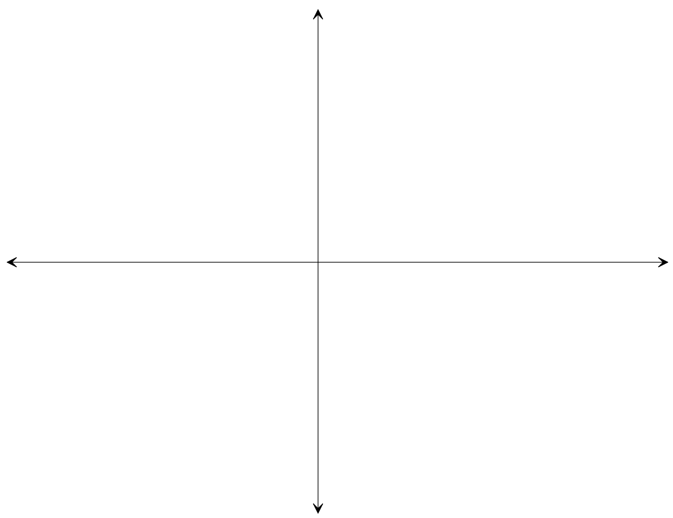
(5)

Question 7: **[20]**

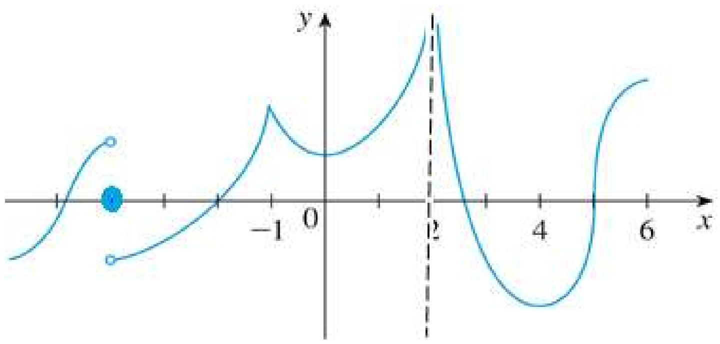
7.1 Sketch the graph of a function that satisfies the given conditions (use the axes provided and show clear readings for the graph):

a) $f(0) = 0$
b) $f'(-2) = f'(1) = 0$
c) $\lim_{x \to 6^-} f(x) = -\infty$
d) $f'(x) > 0$ on $(-2\,,1)$
e) $f'(x) < 0$ on $(-\infty, -2) \cup (1\,,6)$
f) $f''(x) > 0$ on $(-\infty, 0)$
g) $f''(x) < 0$ on $(0\,,6)$

(10)



7.2 Given the graph of the function f, answer the questions below the graph:



a. On what interval(s) of x is f increasing?

(2)

b. On what interval(s) of x is f decreasing?

(2)

c. Why is f not continuous at $x = -4$? Motivate your answer by theory.

(2)

d. Why is f not differentiable at $x = -1$?

(1)

e. Write down the values of the following limits:

$$\lim_{x \to -\infty} f(x) =$$

(1)

$$\lim_{x \to 2} f(x) =$$

(1)

f. Give the equation of the vertical asymptote.

(1)