UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF PURE AND APPLIED MATHEMATICS

MODULE: MAFT0A3/MA3AFET

COURSE: MATHEMATICS 3A FOR TEACHERS

CAMPUS: APK

EXAM: JUNE 2017

DATE: TUESDAY 6 JUNE 2017

TIME: 08:30 – 11:30

ASSESSOR: MS. R. DURANDT

EXTERNAL MODERATOR: DR. B. POSTHUMA

DURATION: 3 HOURS **MARKS:** 120

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SURNAME AND INITIALS

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STUDENT NUMBER

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CONTACT NUMBER

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NUMBER OF PAGES: 16 PAGES (including front page)

INSTRUCTIONS: ANSWER ALL THE QUESTIONS, CALCULATORS ARE NOT ALLOWED

SECTION A: SHORT ANSWER AND THEORY QUESTIONS [11]

Question 1: (5)

Complete the following short answer questions by providing the correct answer in the allocated space.

Question	Answer
Evaluate the integral: $$\int x^{-4}dx$$	
Evaluate the integral: $$\int \frac{dx}{2}$$	
Evaluate the integral: $$\int e^{x}dx$$	
Evaluate the integral: $$\int tanxdx$$	
Evaluate the integral: $$\int \frac{dx}{\sqrt{9-x^2}}$$	

Question 2: (6)

2.1 State the rule for integration by parts.

(1)

2.2 If the expression $\sqrt{a^2 - x^2}$ occurs in an integral, what trigonometry substitution might you try?

(1)

2.3 Write the ratio as a mixed number:

$$\frac{x^5 + x - 1}{x^3 + 1}$$

(2)

2.4 State **True** or **False**:

$$\cos 2x = 2sin^2 x - 1$$

(1)

2.5 State **True** or **False**:

$$\frac{d}{dx}(2^3) = 2^3 ln2$$

(1)

SECTION B: MULTIPLE CHOICE QUESTIONS [18]

Question 3:

The following questions are multiple choice questions. There is only one correct answer from the choices given. Select the correct option by marking the option with a cross (**X**).

3.1 The slope of the tangent line to the graph of $\boldsymbol{y = (\ln x)e^x}$ at the point $x = 2$ is:

a) $\frac{e^2}{2}$
b) e
c) $e^2\left(\ln 2 + \frac{1}{2}\right)$
d) $e(2\ln 2 + 1)$
e) None of the above.

(6)

3.2 Determine the following indefinite integral:

$$\int \frac{\boldsymbol{y^3 + 4y^2}}{\boldsymbol{y^3 + y}}\boldsymbol{dy}$$

a) $\frac{1}{2}y^2 + ln|y^2 + 1| - 2\tan^{-1} y + C$
b) $y + 2ln|y^2 + 1| - \tan^{-1} y + C$
c) $2y^2 + 2ln|y^2 + 1| + \tan^{-1} y + C$
d) $\frac{1}{2}y^2 - 2ln|y^2 + 1| - \frac{1}{4}\tan^{-1} y + C$
e) None of the above.

(6)

3.3 Evaluate the following definite integral:

$$\int_2^4 \frac{\boldsymbol{2dx}}{\boldsymbol{x^2 - 6x + 10}}$$

a) π
b) 0
c) $\frac{\pi}{2}$
d) 2π
e) None of the above.

(6)

SECTION C: APPLICATIONS **[91]**

Question 4: **(15)**

4.1 Each of the regions ***A***, ***B*** and ***C*** bounded by the graph of f and the $x = axis$ has area equal to 3. Find the value of $\int_{-4}^{2}[\boldsymbol{f(x) + 2x + 5}]\,\boldsymbol{dx}$ by using the given sketch below.

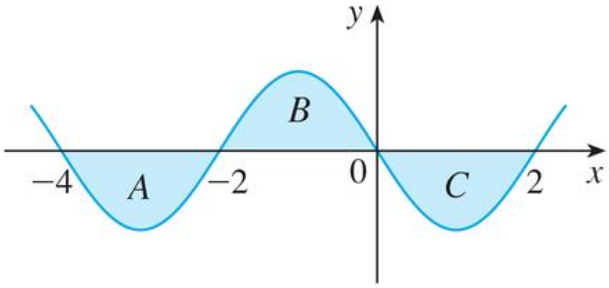

Figure from Stewart, J. (2013). Essential Calculus. CENGAGE Learning.

(5)

4.2 Express the limit as a definite integral on the interval $[1,5]$, but do not solve the integral.

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{x_i}{1+x_i}\Delta x$$

(3)

4.3 Approximate the given definite integral by using a **Riemann sum**, working with four subintervals, taking the sample points to be <u>left</u> endpoints. Explain, by using a diagram (drawing), what the Riemann sum represents and show all calculations.

$$\int_{0}^{2} (x^2 - x)\,dx$$

(7)

Question 5: **(10)**

The half-life of *radium A* is 500 years and a sample of *radium A* has a mass of 10mg. (simplify answers)

5.1 Find a formula for the mass of *radium A* that remains after t years.

(3)

5.2 Find the mass after 1200 years.

(2)

5.3 Find the rate of decay after 3 years.

(3)

5.4 When will the mass be 1mg?

(2)

Question 6: **(20)**

6.1 If $\boldsymbol{g(x) = \int_{1}^{\sqrt{x}} \frac{e^t}{t} dt}$, then calculate $\boldsymbol{g'(1)}$ by using the Fundamental Theorem of Calculus.

(3)

6.2 Calculate the derivative of:

$$\boldsymbol{g(x) = sin[ln(cosx)]}$$

(3)

6.3 Calculate the derivative of:

$$\boldsymbol{y = \cos^{-1}(\sqrt{x}) - x \tan x}$$

(4)

6.4 Calculate the derivative of:

$$\boldsymbol{y = \log_6\left(\frac{1}{x}\right)}$$

(2)

6.5 Calculate the derivative of f and hence find $\boldsymbol{f'(e^3)}$. Given that:

$$\boldsymbol{f(x) = \frac{x}{lnx}}$$

(4)

6.6 Use Logarithmic differentiation to find $\boldsymbol{\frac{dy}{dx}}$ if $\boldsymbol{y = x^{e^{sinx}}}$

(4)

Question 7: **(20)**

7.1 Evaluate the integral:

$$\int x\sqrt{9-5x^2}\,dx$$

(4)

7.2 Evaluate the indefinite integral:

$$\int \frac{x}{1+x^4}\,dx$$

(4)

7.3 Use a proper integration technique to evaluate the integral:

$$\int \boldsymbol{x tan^2 x dx}$$

(6)

7.4 Use a proper integration technique to evaluate the integral:

$$\int \boldsymbol{sin^6 x cos^3 x dx}$$

(6)

Question 8: **(16)**

8.1 Determine the area (A) included by the curves (draw a suitable graph and show clear calculations):

$$\boldsymbol{y = 1 - 2x^2 \;\; and \;\; y = |x|}$$

(8)

8.2 Calculate the volume obtained by rotating the closed region between the curve $\boldsymbol{y = lnx}$ and the lines $\boldsymbol{y = 2,\ \ y = 1}$ and $\boldsymbol{x = 0}$ about the line $y - axis$. Draw a sketch, indicate the cross-sectional area and show all calculations.

(8)

Question 9: **(10)**

9.1 Solve the initial value problem below with $\boldsymbol{y(0) = 2}$, given:

$$\frac{\boldsymbol{dy}}{\boldsymbol{dx}} = \boldsymbol{e^{x-y}}$$

(5)

9.2 Use L'Hôpitals's Rule to obtain the following limits:

$$\lim_{x \to 0^+} (1 - x)^{2/x}$$

(5)