FACULTY OF SCIENCE


STUDENT NUMBER

CONTACT NUMBER

NUMBER OF PAGES: 13 PAGES (including front page)
INSTRUCTIONS: ANSWER ALL THE QUESTIONS, CALCULATORS ARE NOT ALLOWED.

Determine whether the following statements are true or false. If false, explain why or give an example.

| Statement | True or False \& Explanation |
| :---: | :---: |
| The graph of the function is a parabola $y=e^{2} x$ |  |
| The function is symmetrical with respect to the origin $f(x)=\frac{1}{x}$ |  |
| The length of the radius of the given circle is 16 : $(x-1)^{2}+y^{2}=16$ |  |
| The point $(x, y)$ where $x>0$ and $-y>$ 0 lies in the first quadrant |  |
| $\frac{1}{a^{-1}+b^{-1}}=a+b$ |  |

The following questions are multiple choice questions. There is only one correct answer from the choices given. Circle the correct option.
2.1 Which expression is equal to

$$
\left(-c^{2}\right)^{-\frac{1}{3}} ?
$$

A. $\frac{1}{\sqrt[3]{-c^{2}}}$
B. $\frac{1}{\sqrt[3]{c^{2}}}$
C. $\frac{1}{\sqrt{-c^{3}}}$
D. $\sqrt[3]{c^{2}}$
E. None of these
2.2 Use the graph to answer this question

The line $y=\frac{1}{2} x+\frac{1}{2}$ passes through which point?
A. $\quad A(-5 ; 7)$
B. $\quad \mathrm{B}(-7 ;-3)$
C. $\quad C(-7 ; 4)$
D. $\quad D(-5 ;-7)$
E. None of these
2.3 Given the equation $\boldsymbol{a x}+\boldsymbol{b y}+\boldsymbol{c}=\mathbf{0}$, which of the following must be true for the graph of the line to have a positive slope and a positive $\boldsymbol{y}$-intercept?
A. $\quad a>0, b>0, c>0$
B. $\quad a>0, b<0, c>0$
C. $\quad a>0, b>0, c<0$
D. $\quad a>0, b<0, c<0$
E. None of these
2.4 Find the end behaviour of $y=-x^{3}+10 x^{2}-1$
A. $\quad y \rightarrow \infty$ as $x \rightarrow \infty$ and $y \rightarrow \infty$ as $x \rightarrow-\infty$
B. $\quad y \rightarrow \infty$ as $x \rightarrow \infty$ and $y \rightarrow-\infty$ as $x \rightarrow-\infty$
C. $y \rightarrow-\infty$ as $x \rightarrow \infty$ and $y \rightarrow \infty$ as $x \rightarrow-\infty$
D. $\quad y \rightarrow-\infty$ as $x \rightarrow \infty$ and $y \rightarrow-\infty$ as $x \rightarrow-\infty$
E. None of these
2.5 The zeros of $y=x^{3}+4 x^{2}-x-4$ are
A. $\quad-1 ; 1 ; 4$
B. $\quad-1 ; 2 ; 2$
C. $\quad-2 ; 1 ; 2$
D. $-1 ; 1 ;-4$
E. None of these

## Question 3 [14]

3.1 Express as a single logarithm :

$$
3 \log _{2}(x+1)-\log _{2} x^{2}+1
$$

3.2 Express in the form $a+b i$ :

$$
\frac{3-i}{1+2 i}
$$

3.3 Find the equation of the inverse function of $f$.

$$
f(x)=e^{2 x-1}
$$

3.4 Find and simplify $f \circ g$ and find its domain:

$$
f(x)=\frac{2}{x}, \quad g(x)=\frac{1}{x-2}
$$

3.5 Find the quotient and the remainder.

$$
\left(x^{4}+1\right) \div\left(x^{2}-x-1\right)
$$

## Question 4 [19]

4.1 Solve the equation by the method of completing the square:

$$
2 x^{2}+5 x+3=0
$$

4.2 Solve for :
a.

$$
\sqrt{2 x+3}-2=x
$$

b.

$$
e^{2 x}-3 e^{x}+2=0
$$

c.

$$
\log _{3} x+\log _{3}(x-8)=2
$$

d.

$$
\frac{x+1}{x+3} \leq \frac{x-2}{x-1}
$$

## Question $5 \quad$ [12]

5.1 Show that the points are vertices of a right triangle:

$$
A(-2,9), \quad B(1,0) \quad C(-5,3)
$$

5.2 Determine the equation of a circle with centre $(-1,5)$ that passes through the point $(-4,6)$.
5.3 Determine the equation of the straight line through the point $(-1,2)$ that is perpendicular to the line $y=3-2 x$.
5.4 Describe the transformations with respect to the parent function $g(x)=\sqrt{x}$.

$$
f(x)=1-\sqrt{x+2}
$$

## Question 6 <br> [26]

6.1 Sketch the graph of

$$
y=\ln (-1+x)
$$

clearly indicating the asymptote and intercepts with the axes.

### 6.2 Given

$$
f(x)=\left\{\begin{array}{cc}
\sqrt{x-1}, & x>1 \\
2 & x \leq 1
\end{array}\right.
$$

a. Determine $\quad f(-2)$
b. Determine $\quad f(1)$
c. Determine $\quad f(10)$
d. What is the $y$-intercept of $f(x)$ ?
6.3 Given

$$
f(x)=-x^{2}+x+2
$$

a. Express $f$ in the form $f(x)=a(x-h)^{2}+k$
b. Determine the $x$ - and $y$-intercepts of $f(x)$.
c. $\quad$ Sketch the graph of $f(x)$, showing the $x$ - and $y$-intercepts and the turning point.
d. State the range of $f$
e. For which interval(s) is $f$ increasing?

### 6.4 Given

$$
f(x)=\frac{x^{2}-4}{x^{2}}
$$

a. Determine the $x$ - and $y$-intercepts of $f(x)$.
b. Find the vertical asymptote(s) of $f(x)$.
c. Find the horizontal asymptote(s) of $f(x)$.
d. Complete the table:

| $x$ | -3 | -1 | 1 | 3 |
| :---: | :--- | :--- | :--- | :--- |
| $y$ |  |  |  |  |

e. Sketch the graph of $f(x)$.

## Question 7

[9]
7.1 Show that $x+1$ is a factor of $f(x)=x^{3}-3 x-2$
7.2 Use (7.1) to fully factorise $f(x)$.
7.3 Now find the partial fraction decomposition of:

$$
\frac{2 x-7}{x^{3}-3 x-2}
$$

