



FACULTY OF SCIENCE

DEPARTMENT OF PURE AND APPLIED MATHEMATICS

MODULE: MATHEMATICAL ANALYSIS B – MAA00B1
CAMPUS: APK
ASSESSMENT: EXAMINATION

DATE: 27 NOVEMBER 2017
ASSESSORS: MR W VAN REENEN
INTERNAL MODERATOR: MR M POTGIETER
DURATION: 2 HOURS

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INITIALS AND SURNAME: _____

STUDENT NUMBER: _____

CONTACT NUMBER: _____

NUMBER OF PAGES: 14 (INCLUDING COVER PAGE)

INSTRUCTIONS:

- ANSWER ALL THE QUESTIONS IN PEN.
- ALL GRAPHS MUST BE DRAWN IN PEN.
- NO PENCIL OR TIPEX ALLOWED.
- SHOW ALL THE NECESSARY CALCULATIONS CLEARLY.
- IF FORMULAS ARE USED THEY MUST BE STATED AS MARKS ARE GIVEN TO THEM.
- SCIENTIFIC AND FINANCIAL CALCULATORS ARE ALLOWED.
- IF NECESSARY, ROUND OFF TO TWO DECIMAL PLACES.
- THE QUESTIONS CAN BE ANSWERED IN ANY ORDER.

Question 1**[3]**

Given that $f(x) = e^{2x+2}$, determine $f^{-1}(x)$.

Question 2**[6]**

Given the following matrices:

$$A = \begin{bmatrix} 2 & -1 \\ 2 & 4 \end{bmatrix}$$

$$B = \begin{bmatrix} -2 & 1 \\ -2 & 3 \end{bmatrix}$$

$$C = \begin{bmatrix} -1 & 3 \\ 5 & -1 \end{bmatrix}$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

2.1 Determine: $A + 5I$

[2]

2.2 Determine: $A^T - \frac{1}{2}C$

[2]

2.3 Determine: B^3

[2]

Question 3

[5]

Find A^{-1} if $A = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$

Question 4**[9]**

Given the input-output matrix $M = \begin{bmatrix} 200 & 500 & 500 \\ 400 & 200 & 900 \\ 600 & 800 & 0 \end{bmatrix}$

4.1 State the internal demand matrix C . [1]

4.2 State the external demand matrix D . [1]

4.3 Determine the Leontief matrix A . [1]

4.4 Determine the production matrix X . [1]

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- 4.5 If the external demand changes to $D_{new} = \begin{bmatrix} 600 \\ 805 \end{bmatrix}$, determine the new production matrix X_{new} . (Hint: $(I - A)X = D$) [5]

Question 5**[5]**

Differentiate the following function through first principles: $f(x) = \frac{x+3}{x-3}$

{Note: You are not allowed to use the differentiation rules}

Question 6**[8]**

Given the following case-defined function:

$$f(x) = \begin{cases} (x+1)^2 - 1 & \text{if } -3 \leq x < 1 \\ 3 & \text{if } 1 < x < 3 \\ -x + 6 & \text{if } 3 \leq x \leq 6 \end{cases}$$

6.1 Graph the function.

[3]

6.2 Determine:

6.2.1 $\lim_{x \rightarrow 1^-} f(x)$. **[1]**

6.2.2 $\lim_{x \rightarrow 1^+} f(x)$. **[1]**

6.2.3 $\lim_{x \rightarrow 1} f(x)$. **[1]**

6.2.4 $f(1)$. **[1]**

6.3 Is the case defined function continuous at $x = 1$? (**YES** or **NO**) **[1]**

Question 7**[2]**

Determine the following limits:

7.1 $\lim_{x \rightarrow -2} \frac{x^4 - 16}{x + 2}$ [1]

7.2 $\lim_{x \rightarrow -\infty} \frac{8x^4 - 7x^3 - 5x^2 - 2x}{4x^4 + 1000}$ [1]

Question 8**[10]**Differentiate the following functions. You do **NOT** have to simplify your answers.

8.1 $y = \ln \pi^{\log e}$ [1]

8.2 $y = \sqrt[4]{\sqrt[3]{\sqrt{x}}}$ [1]

8.3 $y = \ln \sqrt{3x - 3}$ [1.5]

8.4 $y = 4e^{x^3+x^2+x}$ [1.5]

8.5 $y = \sqrt[3]{5x - 5}$ [2]

8.6 $y = \frac{e^x}{\ln x}$ [3]

Question 9**[4]**

Use implicit differentiation to find the derivative:

$$\ln x \cdot e^y + 2x^2 + 2y^2 = 2$$

Question 10**[5]**

Prove the following differentiation rule:

$$\text{If } f(x) = m(x) \cdot n(x), \text{ then } f'(x) = n(x) \cdot m'(x) + m(x) \cdot n'(x).$$

Question 11**[5]**

Use logarithmic differentiation to find the derivative:

$$y = \sqrt{\frac{x^2 + 2}{x^2 - 2}}$$

Question 12**[4]**

Given the following function:

$$f(x) = (x + 2)^3 - 2$$

12.1 Determine the *domain* of the function.

[1]

12.2 Determine $f'(x)$ [1]

12.3 Determine $f''(x)$ [1]

12.4 Determine the interval where the function is concave up. [1]

Question 13 [9]

Given the following function:

$$f(x) = \frac{2x^2}{x^2 - 1}$$

13.1 Determine the critical points/s of $f(x)$. [4]

13.2 Determine the $f''(x)$. [3]

13.3 Using the second derivative test, determine whether the critical point/s determined in Question 8.1 are a local minimum or local maximum. [2]

Question 14 [7]

Find the indefinite integral:

14.1 [3]

$$\int \frac{e^x + e^{2x}}{e^x} dx$$

14.2

[4]

$$\int (2x + 2)e^{2x^2+4x+1} dx$$

Question 15**[7]**

The demand function for a given product is $p = f(q) = 100 - 0.05q$. The supply function is $p = g(q) = 10 + 0.1q$.

Find the Consumer's Surplus and Producer's Surplus under market equilibrium.

End of Paper – Total 89 Marks

Use this space if you want to redo a question. Clearly indicate at the question that the answer is on Page 14.