UNIVERSITY
JOHANNESBURG

## FACULTY OF SCIENCE

|  | DEPARTMENT OF PURE AND APPLIED MATHEMATICS |  |
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| MODULE: | INTRODUCTORY MATHEMATICAL ANALYSIS A - MAA00A1 \& MAT00A1 |  |
| CAMPUS: | APK |  |
| ASSESSMENT: | SUPPLEMENTARY EXAM |  |
| DATE: | JULY 2017 <br> MSSESSORS: | MR C HATINGIMANA <br> MS ML JUGA <br> MS M NOUKO <br> MR W VAN REENEN |
| INTERNAL MODERATOR: | MR MSW POTGIETER |  |
| DURATION: | 2 HOURS |  |

INITIALS AND SURNAME: $\qquad$

STUDENT NUMBER:

CONTACT NUMBER:

NUMBER OF PAGES: 12 (INCLUDING COVER PAGE)
INSTRUCTIONS:

- ANSWER ALL THE QUESTIONS IN PEN.
- ALL GRAPHS MUST BE DRAWN IN PEN.
- NO PENCIL OR TIPEX ALLOWED.
- SHOW ALL THE NECESSARY CALCULATIONS CLEARLY.
- IF FORMULAS ARE USED THEY MUST BE STATED AS MARKS ARE GIVEN TO THEM.
- SCIENTIFIC AND FINANCIAL CALCULATORS ARE ALLOWED.
- IF NECESSARY, ROUND OFF TO TWO DECIMAL PLACES.
- THE QUESTIONS CAN BE ANSWERED IN ANY ORDER.


## Question 1

Simplify the following expressions completely:
$1.1 \frac{x^{4}-1}{x^{2} y-4 x y+3 y} \div \frac{2 x^{2}+2}{2 x^{2}-3 x-9}$
$1.2 \quad \frac{\left(x^{3}\right)^{2}}{x^{5}} \times\left[\frac{(\sqrt{x})^{12}}{x^{3}}\right]^{2}$

## Question 2

Rationalise the denominator of the following fraction:

$$
\frac{1}{2+\sqrt{y+4}}
$$

## Question 3

Solve for $x$ in the following equations:
$3.1 \ln \left(12 x^{2}-10 x+9\right)=\ln \left(12 x^{2}+19\right)$
$3.2 \quad \frac{e^{(5-2 x)}}{e^{(x+1)}}=e^{10}$

## Question 4

Three cards are drawn from a deck of 52 cards without replacement. Find the probability that:
4.1 All three cards are spades.
4.2 The first card is a spade and the two others are diamonds.
4.3 The first card is a diamond, second card is a spade and the last card is a heart.
4.4 All cards are black.
4.5 All cards are not black.

## Question 6

Given the following system of constraints:

$$
\begin{gathered}
-40 x+20 y \geq-120 \\
-2 x+y \leq 8 \\
y \leq-2 x+10 \\
x, y \geq 0
\end{gathered}
$$

6.1 Graph all of the inequalities on the same set of axes, clearly indicating all intercepts and the feasible region.
6.2 Determine the corner points of the feasible region.
6.3 If possible, determine the value of $x$ and $y$ that will minimize the following function:
[1]

$$
P=0.4 x-0.2 y
$$

## Question 7

Differentiate each of the following:

$$
\begin{equation*}
7.1 \quad f(x)=\ln \left(5 x^{2}+10\right) \tag{2}
\end{equation*}
$$

$7.2 f(x)=\left(\sqrt[3]{39 x-2 x^{2}+9}\right)^{2}$
$7.3 \quad f(x)=\frac{2 x+9}{e^{x-2}}$

## Question 8

Given the consumption function of a country:

$$
C=9+0.8 I^{2}-0.3 \sqrt{I}
$$

8.1 Find the function describing the marginal propensity to consume.
8.2 Find the marginal propensity to consume if $I=25$
8.3 Find the marginal propensity to save if $I=25$.

## Question 9

Given the following function:

$$
f(x)=2 x^{3}-9 x^{2}+12 x+7
$$

9.1 Determine the Domain of the function.
9.2 Determine the coordinate/s of the turning point/s.
9.3 Determine whether the turning point/s are maximum or minimum.
9.4 Determine the intervals along which the function is increasing and decreasing.

### 9.5 Determine the coordinate of the point of inflection.

9.6 Determine the intervals of concavity.

## Question 10

## You are not allowed to use a financial calculator for this question.

Today, Charlotte deposits money into an account which earns 5\% compounded annually for 7 years. How much did she deposit today that yielded her the same amount in the future, as would R2,000 deposited at the end of each year for 7 years at $6 \%$ compounded annually?

Question 11

## You are only allowed to use a financial calculator for this question.

11.1 Given a nominal rate of $7.63 \%$ compounded quarterly, what is the effective rate?

Answer:
11.2 Find the future value of an investment of R1200 at the beginning of each year for 12 years at the rate of $8 \%$ compounded annually.

Answer:
11.3 An initial investment of R10,000 in a business guarantees the following cash flows:

| Year | Cash Flow |
| :---: | :---: |
| 1 | R2,000 |
| 2 | R1,500 |
| 3 | R1,500 |
| 4 | R8,000 |

If we assume an interest rate of 6\% compounded annually, determine the net present value of the cash flows.

Answer:
11.4 South African Airways (SAA) wants to replace one of their Boeing 737-400 aircrafts in 5 years' time with a Boeing 737-800 MAX. SAA estimate that they will be able to sell off their current machine for $\mathrm{R} 400,000,000$ whilst a new machine is estimated at $R 1,000,000,000$. They want to set up a sinking fund for the new purchase, by using the scrap value of their current machine as deposit on the new machine. Pinnacle Industrial Bank offers SAA a savings option, where they will make payments at the start of each month and will earn interest at a rate of $10 \%$, compounded monthly. Determine the required monthly payment into this savings option.

Answer:

End of Assessment - Total 71 Marks

Use this space if you want to redo a question. Clearly indicate at the question that the answer is on Page 12.

