UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF PURE AND APPLIED MATHEMATICS

MODULE: **INTRODUCTORY MATHEMATICAL ANALYSIS A – MAA00A1 and MAT00A1**

CAMPUS: **APK**

ASSESSMENT: **EXAM**

DATE: **30 MAY 2017**

ASSESSORS: **MR C HATINGIMANA**
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DURATION: **2 HOURS**

72

INITIALS AND SURNAME: _____

STUDENT NUMBER: _____

CONTACT NUMBER: _____

NUMBER OF PAGES: **10 (INCLUDING COVER PAGE)**

INSTRUCTIONS:

- **ANSWER ALL THE QUESTIONS IN PEN.**
- **ALL GRAPHS MUST BE DRAWN IN PEN.**
- **NO PENCIL OR TIPEX ALLOWED.**
- **SHOW ALL THE NECESSARY CALCULATIONS CLEARLY.**
- **IF FORMULAS ARE USED THEY MUST BE STATED AS MARKS ARE GIVEN TO THEM.**
- **SCIENTIFIC CALCULATORS ARE ALLOWED.**
- **FINANCIAL CALCULATORS ARE ALLOWED.**
- **IF NECESSARY, ROUND OFF TO TWO DECIMAL PLACES.**
- **THE QUESTIONS CAN BE ANSWERED IN ANY ORDER.**

Question 1 [2]

Rationalise the denominator:

$$\frac{x}{\sqrt{x}+\sqrt{y}}$$

Question 2 [7]

Solve for x:

2.1 $5e^{2x-4}-5=0$ [2]

2.2 $\log(x+2)+\log(x-1)=1$ [5]

Question 3 **[8]**

Given the function: $g(x) = \ln(4x + 1)$

3.1 Complete the following table (correct to **two decimal places**). [3]

x	0.25	0.5	0.75	1	1.25	1.5
$g(x)$						

3.2 Graph $g(x)$ by using the table from Question 3.1. You must clearly label all axes and intercepts. [3]

3.3 What is the domain of the function $g(x)$? [1]

3.4 What is the range of the function $g(x)$? [1]

Question 4 **[11]**

Given the Objective Function:

$$C = 0.2x + 0.3y$$

And the constraints:

$$8x + 12y \geq 24$$
$$12x + 12y \geq 36$$
$$2x + y \geq 4$$
$$x + y \leq 5$$
$$x, y \geq 0$$

4.1 Graph the system of constraints on the same axis-system. Clearly label your axes and Feasible Region. [5]

4.2 Determine all the corner points of the Feasible Region. [5]

4.3 Minimise the Objective Function. [1]

Question 5 [6]

If you draw three cards from a deck one at a time without replacement, what is the probability that:

5.1 All three cards are red? [2]

5.2 You draw a Club, a Heart and a Diamond? [2]

5.3 You don't draw any spades? [2]

Question 6 [3]

A company uses one computer chip in assembling each unit of a product. The chips are purchased from suppliers A, B and C and are randomly picked for assembling a unit. Ten percent come from A, 20% come from B, and the remainder come from C. The probability that a chip from A will prove to be defective in the first 24 hours of use is 0.06, and the corresponding probabilities for B and C are 0.04 and 0.05, respectively.

Draw a complete tree diagram that shows all the possibilities and probabilities.

Question 7 **[9]**

Differentiate the following functions, but **do not simplify your answer**:

7.1 $y = \pi^{\ln(e)}$ [1]

7.2 $f(x) = \sqrt[3]{2x^2 + 4x + 4}$ [2]

7.3 $g(x) = \ln(3x^2 + 6x)$ [1]

7.4 $h(x) = e^{2x^2+4x+4}$ [2]

7.5 $m(x) = \frac{3x^3+9x}{4x^4+16x}$ [3]

Question 8 **[3]**

Given: $f(x) = x^3 - 3x^2 + x - 2$

8.1 Determine the first derivative. [1]

8.2 Determine the second derivative. [1]

8.3 Determine the interval where the function is concave up. [1]

Question 9 **[4]**

Find the *absolute maximum* and/or *absolute minimum* value/s of:

$$f(x) = 100 + (2x + 2)^2 \quad ; \quad -3 \leq x \leq 4$$

Question 10 ***Use of a financial calculator is NOT allowed.*** **[4]**

Inge wants to open a doggy parlour, *Happy Hounds*. She approaches Benjamin as a potential investor. If Benjamin will provide an initial investment of R30,000, Inge will pay Benjamin the following cash flows:

YEAR	CASH FLOW
2	R 10,000
4	R12,000
6	R14,000

Assume an interest rate of 5%, compounded quarterly.

10.1 Determine the net present value (NPV) of the cash flows. [3]

10.2 Is the investment profitable? (**YES** OR **NO**) [1]

Answer:	

Question 11 ***ONLY use a financial calculator for this question.*** **[15]**

11.1 An effective rate of 11.53% per year is equivalent to what nominal rate compounded monthly? [2]

Answer:	

11.2 In 6 years, Samuel's investment of R9,000 grew to R11,750. Determine the interest rate, compounded quarterly, for this investment. [2]

Answer:	

11.3 Xolani and his wife, Thandi, is repaying a home loan of R1,150,000 at 12.5% per year, compounded monthly, with monthly payments over 20 years.

a) Determine the monthly payment. [2]

Answer:	

b) Determine the balance outstanding after the 90^{th} payment. [2]

Answer:	

c) Determine the interest contained in the 210^{th} payment. [2]

Answer:	

d) Determine the total finance charge. [2]

Answer:	

11.4 *South African Airways (SAA)* wants to replace one of their *Boeing 737-400* aircrafts in 5 years' time with a *Boeing 737-800 MAX*. *SAA* estimate that they will be able to sell off their current machine for R 400,000,000 whilst a new machine is estimated at R 1,000,000,000. They want to set up a sinking fund for the new purchase, by using the scrap value of their current machine as deposit on the new machine. *Pinnacle Industrial Bank* offers *SAA* a savings option, where they will make payments at the start of each month and will earn interest at a rate of 10%, compounded monthly. Determine the required monthly payment into this savings option. [3]

Answer:	

End of Assessment – Total 72 Marks

Use this space if you want to redo a question. Clearly indicate at the question that the answer is on Page 10.