UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF PURE AND APPLIED MATHEMATICS

MODULE: **ASME1B1**

COURSE: **APPLICATIONS OF CALCULUS FOR ENGINEERS (ALTERNATIVE SEMESTER)**

CAMPUS: **APK**

EXAM: **JULY SUPPLEMENTARY EXAM 2017**

DATE: **21/07/2017**

TIME: **08:00 – 10:00**

ASSESSOR: **MR W VAN REENEN**

INTERNAL MODERATOR: **DR A CRAIG**

DURATION: **2 HOURS** **MARKS: 70**

SURNAME AND INITIALS

STUDENT NUMBER

CONTACT NUMBER

NUMBER OF PAGES: **1+11 PAGES (including front page)**

INSTRUCTIONS: **ANSWER ALL THE QUESTIONS IN PEN.**

NO CALCULATORS ARE ALLOWED.

If you require extra space, continue on the adjacent blank page next to it and indicate this clearly.

Question 1 [5 marks]

For questions 1.1 - 1.5, choose **one** correct answer, and make a cross (X) in the correct block.

Question	**a**	**b**	**c**	**d**	**e**
1.1					
1.2					
1.3					
1.4					
1.5					

1.1 The average value of $f(x) = x^2\sqrt{x^3+1}$ on the closed interval $[0, 2]$ is: [1]

a) $\frac{26}{9}$

b) $\frac{13}{3}$

c) $\frac{26}{3}$

d) 3

e) None of the above

1.2 Which trigonometric substitution can be used to evaluate the integral $\int \frac{x^3}{\sqrt{x^2+4}}\,dx$? [1]

a) $x = 2\sec\theta$,

b) $x = 2\tan\theta$,

c) $x = 2\cos\theta$,

d) $x = 2\csc\theta$,

e) None of the above

1.3 Write the equation $r = \cos\theta$ using rectangular coordinates. [1]

a) $x^2 + y^2 = y$

b) $(x+y)^2 = x$

c) $x^2 + y^2 = x$

d) $(x+y)^2 = y$

e) None of the above

1.4 Write an equation of the parabola with vertex at the origin and the directrix at $y = 5$. [1]

a) $x = 5y^2$

b) $x = -\frac{1}{20}y^2$

c) $y = -\frac{1}{20}x^2$

d) $y = -20x$

e) None of the above

1.5 If $x = e^{4t}$ and $y = \sin 6t$, then $\frac{dy}{dx} =$ [1]

a) $\dfrac{3e^{-4t}\cos 6t}{2}$

b) $\dfrac{-3\cos 6t}{2e^{4t}}$

c) $\dfrac{3e^{-4t}\cos t}{2}$

d) $3e^{-4t}\cos 6t$

e) None of the above

Question 2 [10 marks]

Given $f(x) = \dfrac{x^2}{x+1}$, find the following:

a) State the domain. [1]

b) Find the intercepts with the axes. [1]

c) Find all asymptotes of $f(x)$, including any slant asymptote(s). [4]

d) Determine the intervals of increase and decrease, as well as the coordinates of the local maximum and/or minimum values if it is given that: $f'(x) = \dfrac{x(x+2)}{(x+1)^2}$ [4]

Question 3 [8 marks]

Evaluate the following integrals:

a) $\displaystyle\int \frac{x^3 - 4x + 1}{x^2 - 3x + 2}\, dx$ [4]

b) $\int \frac{dy}{(9+y^2)^{\frac{3}{2}}}$ [4]

Question 4 [3 marks]

Determine whether or not the integral is convergent or divergent: $\int_3^4 \frac{1}{\sqrt{x-3}}\,dx$ [3]

Question 5 [3 marks]

Solve the following differential equation: $\dfrac{dy}{dx} = \dfrac{y\cos x}{1+y^2}, \quad y(0) = 1$ [3]

Question 6 [3 marks]

Find the length of curve: $f(x) = \left(x - \dfrac{4}{9}\right)^{\frac{3}{2}}, \quad x \in [1, 4]$ [3]

Question 7 [3 marks]

Find the area of the surface generated by revolving the following curve about the x**-axis**.

$$x = y^2 - 18, \;\; 1 \leq y \leq 3$$

[3]

Question 8 [3 mark]

A tank of water in the shape of a cone is leaking water at a constant rate of $2m^3/hour$. The base rate of the tank is $5m$ and the height of the tank is $14m$. HINT: The volume of a cone is represented by: $V = \frac{1}{3}\pi r^2 h$. At what rate is the depth of the water in the tank changing when the depth of the water is $6m$? [3]

Question 9 [4 marks]

Prove Fermat's Theorem, i.e. prove that if f has a local maximum or minimum at c, and if $f'(c)$ exists, then $f'(c) = 0$. [4]

Question 10 [3 marks]

Use the **Binomial Theorem** to expand $(\sqrt{x}-1)^4$. Simplify as far as possible. [3]

Question 11 [5 marks]

a) Find the vertex, focus, directrix and then sketch the curve of $y^2+2y+12x+25=0$. [3]

b) Find an equation of the hyperbola with vertices $(\pm 3, 0)$ and asymptotes $y=\pm 2x$. [2]

Question 12 [4 marks]

Use the **method of cylindrical shells** to find the volume of the solid generated by rotating the region bounded by the following curves, about the line $x = -3$.

$$y = -x^2 + x, \quad y = 0$$

[4]

Question 13 [3 marks]

Sketch the region bounded by the given curves and calculate the area of the region.

$$y = 2x, \;\; y = x^2 - 4x$$

[3]

Question 14 [7 marks]

a) Sketch the curve defined by the parametric equations and indicate the direction with an arrow.

$$x = 4\cos\theta, \quad y = 3\sin\theta, \quad 0 \leq \theta \leq 2\pi$$

[3]

b) Find the slope and concavity at the point $(2, 3)$ for the curve given by parametric equations below .

$$x = \sqrt{t}, \quad y = \frac{1}{4}(t^2 - 4), \quad t \geq 0$$

[4]

Question 15 [6 marks]

Consider the polar equation $r = 1 - 2\sin\theta$

a) Sketch the graph of the given limacon. [3]

b) **Set up an integral** to find the area inside the inner loop of the given limacon. Simplify the integrand as far as possible. [3]