UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF PURE AND APPLIED MATHEMATICS

MODULE: **ASME1B1**

COURSE: **APPLICATIONS OF CALCULUS FOR ENGINEERS (ALTERNATIVE SEMESTER)**

CAMPUS: **APK**

EXAM: **JUNE 2017**

DATE: **30/05/2017**

TIME: **12:30 – 14:30**

ASSESSOR: **MR W VAN REENEN**

INTERNAL MODERATOR: **DR A CRAIG**

DURATION: **2 HOURS** **MARKS: 70**

SURNAME AND INITIALS

STUDENT NUMBER

CONTACT NUMBER

NUMBER OF PAGES: **1+11 PAGES (including front page)**

INSTRUCTIONS: **ANSWER ALL THE QUESTIONS IN PEN.**

NO CALCULATORS ARE ALLOWED.

If you require extra space, continue on the adjacent blank page next to it and indicate this clearly.

Question 1 [5 marks]

For questions 1.1 - 1.5, choose **one** correct answer, and make a cross (X) in the correct block.

Question	**a**	**b**	**c**	**d**	**e**
1.1					
1.2					
1.3					
1.4					
1.5					

1.1 The average of the function $y = \cos x$ on the interval $x \in [-3, 5]$ is: [1]

a) $\dfrac{\sin 5 - \sin 3}{8}$

b) $\dfrac{\sin 5 - \sin 3}{2}$

c) $\dfrac{\sin 5 + \sin 3}{8}$

d) $\dfrac{\sin 5 + \sin 3}{2}$

e) None of the above

1.2 Which trigonometric substitution can be used to evaluate the integral $\displaystyle\int \frac{x^3}{\sqrt{x^2-4}}\,dx$? [1]

a) $x = 2\sec\theta$

b) $x = 2\tan\theta$

c) $x = 2\cos\theta$

d) $x = 2\csc\theta$

e) None of the above

1.3 Write the equation $r = 10\sin\theta$ using rectangular coordinates. [1]

a) $\sqrt{x^2 + y^2} = 10y$

b) $x^2 + y^2 = 10y$

c) $\sqrt{x^2 + y^2} = 10x$

d) $x^2 + y^2 = 10y$

e) None of the above

1.4 Write an equation of the parabola with vertex at the origin and focus at $(-2, 0)$. [1]

a) $x = -\frac{1}{8}y^2$

b) $x = -\frac{1}{4}x^2$

c) $x = \frac{1}{8}x^2$

d) $x = \frac{1}{8}y^2$

e) None of the above

1.5 Consider the differential equation $xy' - 2y = x^2$ where $x > 0$. The integrating factor $I(x)$ is: [1]

a) $\frac{1}{x^2}$
b) $e^{\ln x^2}$
c) x^2
d) $e^{2 \ln x}$
e) None of the above

Question 2 [9 marks]

Given $f(x) = \dfrac{x^2 - 3}{x^3}$, find the following:

a) Intercepts with the x-axis and y-axis. [2]

b) Asymptotes [2]

c) Interval of increase and decrease. [3]

d) Local maximum and minimum values. [2]

Question 3 [7 marks]

Evaluate the following integrals:

a) $\int \frac{3x+11}{x^2-x-6}\,dx$ [3]

b) $\int e^x \cos x \, dx$ [4]

Question 4 [3 marks]

Determine whether the following integral is convergent or divergent: $\int_0^\infty \frac{x}{(x^2+2)^2} \, dx$. [3]

Question 5 [4 marks]

Solve the following differential equation: $\frac{dy}{dx}\cos^2(x) + y - 1 = 0, \;\; y(0) = 5$ [4]

Question 6 [3 marks]

Find the length of curve: $y = \ln(\cos x), \;\; 0 \leq x \leq \frac{\pi}{3}$ [3]

Question 7 [4 marks]

Find the area of the surface generated by revolving the following curve about the x**-axis**.

$$y = 2\sqrt{1-x}, \quad x \in [-1, 0]$$

[4]

Question 8 [3 marks]

Air is being pumped into a spherical balloon at a rate of $5cm^3/min$. Determine the rate at which the radius of the balloon is increasing when the diameter of the balloon is $20cm$. [HINT: The volume of a sphere is given by $V = \frac{4}{3}\pi r^3$] [3]

Question 9 [4 marks]

Prove Rolle's Theorem, i.e. prove that if a function f satisfies the following hypothesis:
1. f is continuous on the closed interval $[a, b]$,
2. f is differentiable on the open interval (a, b),
3. $f(a) = f(b)$,
then there is a number c in (a, b) such that $f'(c) = 0$. [4]

Question 10 [3 marks]

Use the Binomial Theorem to expand $(x - \sqrt{2})^5$. Simplify as far as possible. [3]

Question 11 [5 marks]

a) Find the vertex, focus, directrix and sketch the conic section: $(x + 2)^2 = 8(y - 3)$. [3]

b) Find an equation of the ellipse with foci $(\pm 2, 0)$ and vertices $(\pm 5, 0)$. [2]

Question 12 [4 marks]

Use the **disk/washer method** to find the volume of the solid generated by rotating the region bounded by the following curves about the x-axis.

$$y = 2x^2, \quad y = x + 1, \quad x \geq 0$$

[4]

Question 13 [3 marks]

Sketch the region bounded by the given curves and **set up an integral** to calculate the area of the region. Simplify the integrand as far as possible.

$$y = 4x + 16, \quad y = 2x^2 + 10$$

[3]

Question 14 [6 marks]

a) Sketch the curve defined by the parametric equations and indicate the direction with an arrow.

$$x = \ln t, \quad y = \sqrt{t}, \quad t \geq 1$$

[3]

b) Find an equation of the tangent line to the curve given by parametric equations below at $t = \frac{\pi}{4}$.

$$x = \sec t, \quad y = \tan t, \quad -\frac{\pi}{2} < t < \frac{\pi}{2}$$

[3]

Question 15 [7 marks]

Consider the polar equation $r = 2 + 2\cos\theta$.

a) Sketch the graph of the given cardioid. [3]

b) **Set up an integral** to find the area inside the given cardioid and outside $r = 3$. Simplify the integrand as far as possible. [4]