UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF PURE AND APPLIED MATHEMATICS

MODULE: **ASME1A1**

COURSE: **CALCULUS OF ONE VARIABLE FUNCTIONS FOR ENGINEERS**

(ALTERNATIVE SEMESTER)

CAMPUS: **APK**

EXAM: **NOVEMBER 2017**

DATE: **20/11/2017**

TIME: **16:30 – 18:30**

ASSESSOR: **MR W VAN REENEN**

INTERNAL MODERATOR: **DR A CRAIG**

DURATION: **2 HOURS** **MARKS: 70**

SURNAME AND INITIALS

STUDENT NUMBER

CONTACT NUMBER

NUMBER OF PAGES: **1+11 PAGES (including front page)**

INSTRUCTIONS: **ANSWER ALL THE QUESTIONS IN PEN**

NO CALCULATORS ALLOWED.

If you require extra space, continue on the adjacent blank page next to it and indicate this clearly.

Question 1 [10 marks]

For questions 1.1 - 1.10, choose **one** correct answer, and make a cross (X) in the correct block.

Question	**a**	**b**	**c**	**d**	**e**
1.1					
1.2					
1.3					
1.4					
1.5					
1.6					
1.7					
1.8					
1.9					
1.10					

1.1 Convert 540° to radians. [1]

a) $\frac{3\pi}{2}$

b) $\frac{2\pi}{3}$

c) $\frac{3\pi}{1}$

d) $\frac{1}{3\pi}$

e) None of the above

1.2 Convert the complex number $z = \sqrt{2} - \sqrt{2}i$ into polar form. [1]

a) $2(\cos(-\frac{\pi}{4}) - i\sin(-\frac{\pi}{4}))$

b) $2(\cos(-\frac{7\pi}{4}) - i\sin(-\frac{7\pi}{4}))$

c) $2(\cos(\frac{\pi}{4}) - i\sin(\frac{\pi}{4}))$

d) $\sqrt{2}(\cos(-\frac{\pi}{4}) - i\sin(-\frac{\pi}{4}))$

e) None of the above

1.3 The correct expansion of $\sum_{i=3}^{6} \frac{5^{i-1}}{i-1}$ is: [1]

a) $\frac{5}{1} + \frac{5^2}{2} + \frac{5^3}{3} + \frac{5^4}{4}$

b) $\frac{5^2}{2} + \frac{5^3}{3} + \frac{5^4}{4} + \frac{5^5}{5}$

c) $\frac{5^0}{1} + \frac{5^1}{1} + \frac{5^2}{2} + \frac{5^3}{3}$

d) $\frac{5^{-1}}{1} + \frac{5^0}{2} + \frac{5^1}{3} + \frac{5^2}{4}$

e) None of the above

1.4 Find f such that $(f \circ g) = F$ given that $g(x) = \sqrt{x} + 1$ and $F(x) = x + 2\sqrt{x}$. [1]

a) $f(x) = x^2 + 2$

b) $f(x) = x^2 - 1$

c) $2(x^2 - 1)$

d) $f(x) = (\sqrt{x} + 1) + 2(\sqrt{\sqrt{x} + 1})$

e) None of the above

1.5 Solve for x in $|3 - 4x| \geq 2$ [1]

a) $x < \frac{3}{2}$ and $x > 0$

b) $x \geq \frac{1}{4}$ and $x \leq \frac{5}{4}$

c) $x < 1$ and $x > \frac{2}{3}$

d) $x \leq \frac{1}{4}$ and $x \geq \frac{5}{4}$

e) None of the above

1.6 Find the domain of the function $f(x) = \dfrac{1}{\sqrt{x} - 1}$ [1]

a) $(-\infty, 0) \cup (0, 1]$

b) $(-\infty, 0) \cup (0, 1) \cup (1, \infty)$

c) $[0, 1) \cup (1, \infty)$

d) $(-\infty, 1) \cup (1, \infty)$

e) None of the above

1.7 If $f(x) = \sin(e^{-x})$, then $f'(x) =$ [1]

a) $-\cos(e^{-x})$

b) $\cos(e^{-x}) - e^{-x}$

c) $e^{-x}\cos(e^{-x})$

d) $-e^{-x}\cos(e^{-x})$

e) None of these

1.8 If f and g are twice differentiable and if $h(x) = f(g(x))$, then $h''(x) =$ [1]

a) $f''(g(x))g'(x) + f'(g(x))g''(x)$

b) $f''(g(x))[g'(x)]^2$

c) $f''(g(x))[g'(x)]^2 + f'(g(x))g''(x)$

d) $f''(g(x))g''(x)$

e) None of these

1.9 Which of the following functions is odd: [1]

a) $f(x) = x^2$

b) $f(x) = \sin x$

c) $f(x) = \cos x$

d) $f(x) = e^x$

e) None of these

1.10 Which of the following expressions is logically equivalent to $p \to q$? [1]

a) $p \vee q$

b) $\neg q \to \neg p$

c) $\neg q \to p$

d) $\neg p \wedge q$

e) None of these

Question 2 [5 marks]

In the table below, Column A contains logical formulas and column B contains descriptions of the formulas. Match each formula in Column A to its description in Column B. [5]

Column A	Column B
(i) $(p \wedge \neg q) \vee (q \wedge \neg p)$	(a) A tautology
(ii) $p \vee \neg p$	(b) Logically equivalent to $\neg p \wedge q$
(iii) $q \rightarrow p$	(c) A contradiction
(iv) $p \wedge \neg p$	(d) The converse of $p \rightarrow q$
(v) $\neg p \leftrightarrow \neg q$	(e) Logically equivalent to $(p \leftrightarrow q)$

Answers

Column A	Column B
(i)	
(ii)	
(iii)	
(iv)	
(v)	

Question 3 [5 marks]

a) State the Intermediate Value Theorem. [2]

b) Use the Intermediate Value Theorem to show that the function $f(x) = 2x^5 - 3x^3 + x - 1$ has a root in the interval $(1, 2)$. [3]

Question 4 [6 marks]

a) Solve for x if: $\dfrac{2x-2}{x-1} > -\dfrac{x+5}{x-1}$ [2]

b) Sketch the graph of $\sec x$ for $x \in \left[-\frac{3\pi}{2}, \frac{3\pi}{2}\right]$. Label all intercepts with axes as well as any asymptotes. [2]

c) Let $z = 6 + 3i$ and $w = 1 - i$. Find $\frac{z}{w}$ and write your answer in the form $a + bi$. [2]

Question 5 [4 marks]

Determine (without using L'Hospital's Rule):

a) $\lim\limits_{x \to 3} \dfrac{x^4 - 81}{x - 3}$ [2]

b) $\lim_{x\to\infty} \dfrac{1-e^x}{1+2e^x}$ [2]

Question 6 [5 marks]

Evaluate the following limits (use L'Hospital's Rule when needed):

a) $\lim_{x\to-4} \dfrac{\sin \pi x}{x^2-16}$ [2]

b) $\lim_{x\to\infty} [e^x + x]^{\frac{1}{x}}$ [3]

Question 7 [8 marks]

Given the following case-defined function:

$$f(x) = \begin{cases} (x+1)^2 - 1 & \text{if } -3 \leq x < 1 \\ 2^x + 1 & \text{if } 1 < x < 3 \\ -x + 12 & \text{if } -3 \geq -x \geq -12 \end{cases}$$

a) Sketch the graph of $f(x)$. [3]

Use the graph to answer the following limit questions:

b) Determine $\lim_{x \to 1^-} f(x)$. [1]

c) Determine $\lim_{x \to 1^+} f(x)$. [1]

d) Determine $\lim_{x \to 1} f(x)$. [1]

e) Determine $f(1)$. [1]

f) Is the case defined function continuous at $x = 1$? Motivate your answer. [1]

Question 8 [7 marks]

Differentiate the following functions:

a) $f(x) = \tan x + x^2 \operatorname{csch}(x)$ [1]

b) $xy^2 + \sqrt{xy} = 2$ [3]

c) $y = \dfrac{\cos^2 3x}{1 + 2e^x}$ [3]

Question 9 [6 marks]

Use the result $\lim_{\theta \to 0} \dfrac{\sin\theta}{\theta} = 1$ to prove the following results:

a) $\lim_{\theta \to 0} \dfrac{\cos\theta - 1}{\theta} = 0$ [3]

b) $\dfrac{d}{dx}(\sin x) = \cos x$, using the definition of the derivative of the function. [3]

Question 10 [5 marks]

a) Find a counter example of the statement: "The product of any two irrational numbers is irrational". [2]

b) Use proof by contradiction to prove the following:
"Let $n \in \mathbb{Z}$, if n^2 is even, then n is even." [3]

Question 11 [2 marks]

State the Fundamental Theorem of Calculus (Part 1 and 2). [2]

Question 12 [7 marks]

Evaluate the following integrals

a) $\int_0^{\frac{\pi}{4}} \sec^2\theta \; d\theta$ [2]

b) $\int_1^4 \sqrt{t}(1+t)\ dt$ [2]

c) $\int e^x \cos(e^x)\ dx$ [3]