APPLIED MATHEMATICS

Vector Algebra and Vector Geometry
APM1EB1/APM1A1E

November Examination: 13/11/2017

Duration: 120 Minutes
Assessor: Mr JM Homann
Moderator: Dr GJ Kemp

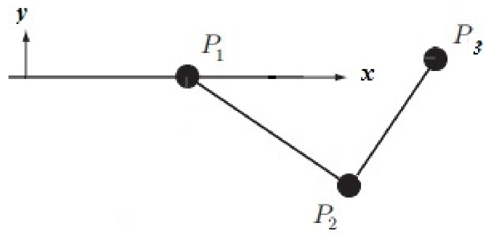
Marks: 50
Session: 08H30-11H30
Venue: D1 Lab K08

Instructions:

1. Answer all of the questions. All calculations must be shown.
2. All symbols have their usual meaning.
3. All angles are measured in degrees.
4. Only answers written in ink will be marked. Do not use red ink.
5. You may answer the questions in any order, but clearly indicate the question number. Rule off after each question. Furthermore, if you answer a particular question more than once, then scratch out the ones that must not be marked and clearly indicate which one must be marked. If this is not done, then one of the versions of the question will be marked, by choice of the marker, without complaint from the exam taker.
6. **No books or notes are allowed.**
7. You will be penalised for using incorrect notation.
8. You will be penalised if you do not use the same notation as described in each question.
9. The use of pocket calculators is permitted. Only round off your final answer (to three decimal places). No marks will be awarded for using rounded off answers in intermediate steps and no marks will be awarded for an incorrect final answer due to rounding errors during calculations.
10. Only use vector algebraic methods for solving the problems. Do not assume any geometrical results which are not given in a problem. You may, however, assume the result of V1.6.3 as a given.
11. This question paper consists of 2 pages, including this one.

Question 1 (10 marks)
Consider three identical masses confined to the x-y-plane; one at P_1, one at P_2 and one at P_3 (see the figure below). The masses are connected by rigid rods, each with a fixed length a. Each rod is free to rotate clockwise or counter-clockwise. P_1 is restricted to move along the x-axis.

1. Reproduce the figure in your answer sheet an indicate the set G of generalised co-ordinates on the figure and then write G as a set.
2. Write the x-y position of each point P_1, P_2 and P_3 by using the co-ordinates of G.



Question 2 (5 marks)
Using only graphical vector geometric techniques, prove that the vector sum is associative.

Question 3 (10 marks)
In $\triangle OAB$, $\overline{OA} = 8\overline{a}$ and $\overline{OB} = 8\overline{b}$. The midpoint of OA is M and the point P lies on AB such that $AP : PB = 3 : 1$. The midpoint of OP is N.

1. Calculate, in terms of $\overline{a}$ and $\overline{b}$, the vectors $\overline{AB}, \overline{OP}, \overline{ON}$ and $\overline{MN}$.
2. The line AN meets OB at C if it is continued. Given that $\overline{OC} = k\overline{b}$, determine the value of k.

Question 4 (5 marks)
M is the midpoint of the line between $A\,(6, -5, 0)$ and $B\,(4, -3, 2)$. Show that M is co-linear with $C\,(2, -6, 0)$ and $D\,(11, 0, 3)$.

Question 5 (10 marks)
The methane molecule (CH_4) has a tetrahedral configuration: the C-atom is at the centre of a cube and the four H-atoms are situated at corners of the cube in such a way that the distances between them are maximal. Find the angle subtended at the C-atom by any two H-atoms.

Question 6 (5 marks)
Solve the following equation for $\overline{r}$:

$$\overline{r} \cdot \overline{a} = \alpha, \quad \overline{r} \times \overline{b} = \overline{c} \quad \text{and} \quad \overline{a} \cdot \overline{b} \neq 0.$$

You may find the following identity helpful:
$\overline{d} \times \left(\overline{e} \times \overline{f}\right) = \left(\overline{d} \cdot \overline{f}\right)\overline{e} - \left(\overline{d} \cdot \overline{e}\right)\overline{f}$.

Question 7 (5 marks)
Show that

$$\left(\overline{a} \times \overline{b}\right) \times \left(\overline{b} \times \overline{c}\right) = \left(\overline{a} \cdot \overline{b} \times \overline{c}\right)\overline{b}.$$

END OF QUESTION PAPER