UNIVERSITY
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JOHANNESBURG

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| **PROGRAM** | BACCALAUREUS TECHNOLOGIAE<br>*CHEMICAL ENGINEERING* |
| **SUBJECT** | **PROCESS CONTROL IV** |
| **CODE** | **ICP411** |
| **DATE** | : SUMMER EXAMINATION 2017<br>23 NOVEMBER 2017 |
| **DURATION** | : (SESSION 1) 08:30 - 11:30 |
| **TOTAL MARKS** | 100 |
| **FULL MARKS** | 100 |

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| **EXAMINER** | DR T. MASHIFANA AND DR N MAZANA |
| **MODERATOR** | DR T.A. MAMVURA |
| **NUMBER OF PAGES** | FIVE (7) INCLUDING THIS COVER PAGE AND ANNEXURES |

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**INSTRUCTIONS**

THIS IS A CLOSED BOOK EXAM
NON-PROGRAMMABLE CALCULATORS PERMITTED (ONLY ONE PER CANDIDATE)
SHOW ALL UNITS IN CALCULATIONS!!!
ANSWER ALL THE QUESTIONS
NO ELECTRONIC DEVICES ALLOWED.

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## QUESTION 1

A stream containing a solute (e.g. sugar) is diluted with water in a well stirred tank. The concentration in the tank is uniform and therefore the stream leaving the tank has the same concentration at any given time. A special balance on the solute gives:

$$q_i c_i - (q_w + q_i)c = \frac{d(Vc)}{dt} = V\frac{dc}{dt}$$

where c and ci are concentrations, mass/volume.

1.1. Determine the transfer function of this first-order system (20)

1.2. Determine τp and Kp (5)

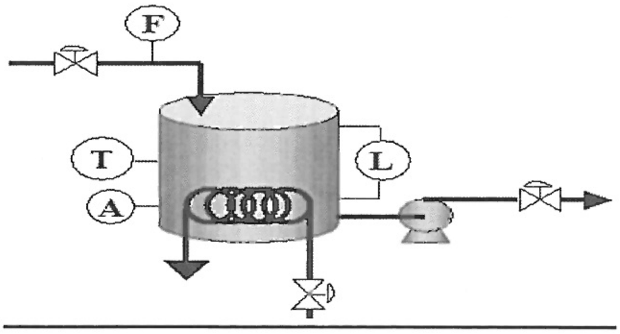
**[25]**

## QUESTION 2

A stirred tank heater is shown in Figure 2.1. The objectives is to maintain the level of the liquid in the tank at level $H_s$ (desired height), to maintain the temperature inside the tank at $T_s$.(desired temperature) and to maintain the flow rate of the liquid into the tank at Fs (desired flow rate).

2.1. Use a feedback control configuration to design a control system to meet the specified objectives by following the control system design steps. (20)

***Note: Do not use different diagrams; provide the design in one diagram***




**F = flow**

**L = level**

**P = pressure**

**T = temperature**

**.....**

Figure 2.1 CSTR

**[20]**

## QUESTION 3:

3.1. Determine the Laplace transforms of the following functions from provided Laplace transform Tables:

a) $f(t)=8$ (3)

b) $f(t)=1-e^{(-t/a)}$ (5)

c) $f(t)=\sin 3t$ (5)

3.2. Draw a generalized closed loop feedback system with a controller block, a final control element block, a process block and measurement block. The system is affected by an external disturbance. Derive the generalized feedback transfer function and state the characteristic equation. (12)

**[25]**

## QUESTION 4

You are given a feedback system. The process with $G_p=1/(s^2+2s+2)$ is controlled by a PI controller i.e. $G_c=K_c\ (1+1/\tau_I\ )$. You are provided with the following additional information:

$G_f=1$; $G_m=1$; $G_d=1/(3s+1)$

Suggest the characteristic equation on the basis of the available block transfer functions. (5)

Check the stability of the system using the Routh _Hurwitz method when given that

$K_c=100$ and $\tau_I=0.1$ (25)

**[30]**

**TOTAL MARKS = 100**

**FULL MARKS = 100**

## Table of Laplace Transforms

| | $f(t)=\mathcal{L}^{-1}\{F(s)\}$ | $F(s)=\mathcal{L}\{f(t)\}$ | | $f(t)=\mathcal{L}^{-1}\{F(s)\}$ | $F(s)=\mathcal{L}\{f(t)\}$ |
|---|---|---|---|---|---|
| 1. | $1$ | $\frac{1}{s}$ | 2. | $\mathbf{e}^{at}$ | $\frac{1}{s-a}$ |
| 3. | $t^n,\quad n=1,2,3,\ldots$ | $\frac{n!}{s^{n+1}}$ | 4. | $t^p,\ p>-1$ | $\frac{\Gamma(p+1)}{s^{p+1}}$ |
| 5. | $\sqrt{t}$ | $\frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$ | 6. | $t^{n-\frac{1}{2}},\quad n=1,2,3,\ldots$ | $\frac{1\cdot3\cdot5\cdots(2n-1)\sqrt{\pi}}{2^n s^{n+\frac{1}{2}}}$ |
| 7. | $\sin(at)$ | $\frac{a}{s^2+a^2}$ | 8. | $\cos(at)$ | $\frac{s}{s^2+a^2}$ |
| 9. | $t\sin(at)$ | $\frac{2as}{\left(s^2+a^2\right)^2}$ | 10. | $t\cos(at)$ | $\frac{s^2-a^2}{\left(s^2+a^2\right)^2}$ |
| 11. | $\sin(at)-at\cos(at)$ | $\frac{2a^3}{\left(s^2+a^2\right)^2}$ | 12. | $\sin(at)+at\cos(at)$ | $\frac{2as^2}{\left(s^2+a^2\right)^2}$ |
| 13. | $\cos(at)-at\sin(at)$ | $\frac{s\left(s^2-a^2\right)}{\left(s^2+a^2\right)^2}$ | 14. | $\cos(at)+at\sin(at)$ | $\frac{s\left(s^2+3a^2\right)}{\left(s^2+a^2\right)^2}$ |
| 15. | $\sin(at+b)$ | $\frac{s\sin(b)+a\cos(b)}{s^2+a^2}$ | 16. | $\cos(at+b)$ | $\frac{s\cos(b)-a\sin(b)}{s^2+a^2}$ |
| 17. | $\sinh(at)$ | $\frac{a}{s^2-a^2}$ | 18. | $\cosh(at)$ | $\frac{s}{s^2-a^2}$ |
| 19. | $\mathbf{e}^{at}\sin(bt)$ | $\frac{b}{(s-a)^2+b^2}$ | 20. | $\mathbf{e}^{at}\cos(bt)$ | $\frac{s-a}{(s-a)^2+b^2}$ |
| 21. | $\mathbf{e}^{at}\sinh(bt)$ | $\frac{b}{(s-a)^2-b^2}$ | 22. | $\mathbf{e}^{at}\cosh(bt)$ | $\frac{s-a}{(s-a)^2-b^2}$ |
| 23. | $t^n\mathbf{e}^{at},\quad n=1,2,3,\ldots$ | $\frac{n!}{(s-a)^{n+1}}$ | 24. | $f(ct)$ | $\frac{1}{c}F\left(\frac{s}{c}\right)$ |
| 25. | $u_c(t)=u(t-c)$ <br> Heaviside Function | $\frac{\mathbf{e}^{-cs}}{s}$ | 26. | $\delta(t-c)$ <br> Dirac Delta Function | $\mathbf{e}^{-cs}$ |
| 27. | $u_c(t)f(t-c)$ | $\mathbf{e}^{-cs}F(s)$ | 28. | $u_c(t)g(t)$ | $\mathbf{e}^{-cs}\mathcal{L}\{g(t+c)\}$ |
| 29. | $\mathbf{e}^{ct}f(t)$ | $F(s-c)$ | 30. | $t^n f(t),\quad n=1,2,3,\ldots$ | $(-1)^n F^{(n)}(s)$ |
| 31. | $\frac{1}{t}f(t)$ | $\int_s^\infty F(u)\,du$ | 32. | $\int_0^t f(v)\,dv$ | $\frac{F(s)}{s}$ |
| 33. | $\int_0^t f(t-\tau)g(\tau)\,d\tau$ | $F(s)G(s)$ | 34. | $f(t+T)=f(t)$ | $\frac{\int_0^T \mathbf{e}^{-st}f(t)\,dt}{1-\mathbf{e}^{-sT}}$ |
| 35. | $f'(t)$ | $sF(s)-f(0)$ | 36. | $f''(t)$ | $s^2F(s)-sf(0)-f'(0)$ |
| 37. | $f^{(n)}(t)$ | $s^nF(s)-s^{n-1}f(0)-s^{n-2}f'(0)\cdots-sf^{(n-2)}(0)-f^{(n-1)}(0)$ | | | |

## Table Notes

1. This list is not a complete listing of Laplace transforms and only contains some of the more commonly used Laplace transforms and formulas.

2. Recall the definition of hyperbolic functions.

$$\cosh(t)=\frac{\mathbf{e}^t+\mathbf{e}^{-t}}{2} \qquad \sinh(t)=\frac{\mathbf{e}^t-\mathbf{e}^{-t}}{2}$$

3. Be careful when using "normal" trig function vs. hyperbolic functions. The only difference in the formulas is the "+ $a^2$" for the "normal" trig functions becomes a "- $a^2$" for the hyperbolic functions!

4. Formula #4 uses the Gamma function which is defined as

$$\Gamma(t)=\int_0^\infty \mathbf{e}^{-x}x^{t-1}\,dx$$

If $n$ is a positive integer then,

$$\Gamma(n+1)=n!$$

The Gamma function is an extension of the normal factorial function. Here are a couple of quick facts for the Gamma function

$$\Gamma(p+1)=p\Gamma(p)$$

$$p(p+1)(p+2)\cdots(p+n-1)=\frac{\Gamma(p+n)}{\Gamma(p)}$$

$$\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$$

TABLE 8.1

INVERSE LAPLACE TRANSFORMS OF SELECTED EXPRESSIONS

| Laplace transform: $\bar{f}(s)$ | Time function: $f(t)$ |
|---|---|
| 1. $\dfrac{1}{(s+a)(s+b)}$ | $\dfrac{e^{-at}-e^{-bt}}{b-a}$ |
| 2. $\dfrac{1}{(s+a)(s+b)(s+c)}$ | $\dfrac{e^{-at}}{(b-a)(c-a)}+\dfrac{e^{-bt}}{(c-b)(a-b)}+\dfrac{e^{-ct}}{(a-c)(b-c)}$ |
| 3. $\dfrac{s+a}{(s+b)(s+c)}$ | $\dfrac{1}{c-b}[(a-b)e^{-bt}-(a-c)e^{-ct}]$ |
| 4. $\dfrac{a}{(s+b)^2}$ | $ate^{-bt}$ |
| 5. $\dfrac{a}{(s+b)^3}$ | $\dfrac{a}{2}t^2e^{-bt}$ |
| 6. $\dfrac{a}{(s+b)^{n+1}}$ | $\dfrac{a}{n!}t^n e^{-bt}$ |
| 7. $\dfrac{1}{s(as+1)}$ | $1-e^{-t/a}$ |
| 8. $\dfrac{1}{s(as+1)^2}$ | $1-\dfrac{a+t}{a}e^{-t/a}$ |
| 9. $\dfrac{\omega^2}{s(s^2+2\zeta\omega s+\omega^2)}$ | $1+\dfrac{e^{-\zeta\omega t}}{\sqrt{1-\zeta^2}}\sin(\omega\sqrt{1-\zeta^2}\,t-\phi)$ where $\cos\phi=-\zeta$ |
| 10. $\dfrac{s}{(1+as)(s^2+\omega^2)}$ | $-\dfrac{1}{1+a^2\omega^2}e^{-t/a}+\dfrac{1}{\sqrt{1+a^2\omega^2}}\cos(\omega t-\phi)$ where $\phi=\tan^{-1}a\omega$ |
| 11. $\dfrac{s}{(s^2+\omega^2)^2}$ | $\dfrac{1}{2\omega}t\sin\omega t$ |
| 12. $\dfrac{1}{(s+a)[(s+b)^2+\omega^2]}$ | $\dfrac{e^{-at}}{(a-b)^2+\omega^2}+\dfrac{e^{-bt}\sin(\omega t-\phi)}{\omega[(a-b)^2+\omega^2]^{1/2}}$ where $\phi=\tan^{-1}\left(\dfrac{\omega}{a-b}\right)$ |

# The Routh Array

| | | | | | |
|---|---|---|---|---|---|
| Row 1 | $a_0$ | $a_2$ | $a_4$ | $a_6$ | ........ |
| 2 | $a_1$ | $a_3$ | $a_5$ | $a_7$ | ........ |
| 3 | $A_1$ | $A_2$ | $A_3$ | | ........ |
| 4 | $B_1$ | $B_2$ | $B_3$ | | ........ |
| 5 | $C_1$ | $C_2$ | $C_3$ | | |
| ....... | ......... | ......... | ......... | ......... | ......... |

Where:

$A_1 = \frac{a_1 a_2 - a_0 a_3}{a_1}$; $A_2 = \frac{a_1 a_4 - a_0 a_5}{a_1}$; $A_3 = \frac{a_1 a_6 - a_0 a_7}{a_1}$ ........

$B_1 = \frac{A_1 a_3 - a_1 A_2}{A_1}$; $B_2 = \frac{A_1 a_5 - a_1 A_3}{A_1}$ .........

$C_1 = \frac{B_1 A_2 - A_1 B_2}{B_1}$; $C_2 = \frac{B_1 A_3 - A_1 B_3}{B_1}$ .........

etc.