UNIVERSITY OF JOHANNESBURG

PROGRAM	:	NATIONAL DIPLOMA **MECHANICAL ENGINEERING**
SUBJECT	:	**APPLIED STRENGTH OF MATERIALS 3**
CODE	:	**ASM 301**
DATE	:	MAIN EXAMINATION 20 NOVEMBER, 2017
DURATION	:	(SESSION 1) 08:30 – 11:30 HRS
WEIGHT	:	40:60
TOTAL MARKS	:	90
FINAL MARKS	:	100

ASSESSOR	:	A. MASHAMBA
MODERATOR	:	K SITHOLE
NUMBER OF PAGES	:	5 PAGES + 5 ANNEXURE

INSTRUCTIONS

1. QUESTIONS IN THIS EXAM PAPER ASSESS APPLICATION OF ENGINEERING KNOWLEDGE (ELO 2).
2. ANSWER ALL QUESTIONS AND SHOW ALL CALCULATIONS.
3. ANSWERS WITHOUT UNITS WILL BE IGNORED.
4. All DIMENSIONS ARE IN mm UNLESS STATED OTHERWISE.
5. FOR MISSING DATA, ASSUME TYPICAL ENGINEERING VALUES.

QUESTION 1

A simply supported steel beam carries two point loads of 10 kN and 20 kN and a uniformly distributed load of 20 kN/m as shown in Figure Q1. The steel beam has a length of 3 m and a flexural rigidity (EI) of 6 MNm^2.

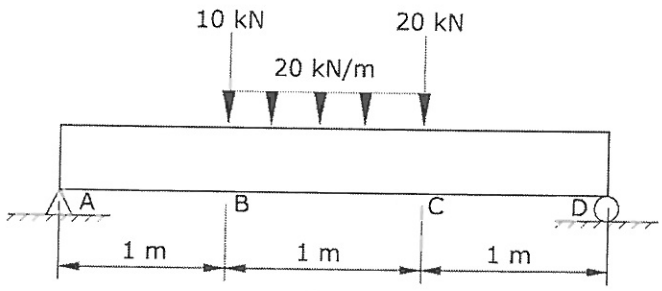


Figure Q1

Assuming that the steel beam has a negligible weight, calculate:

a) the reactions at Point A and D and; (4)

b) the maximum deflection of the beam. (12)

[16]

QUESTION 2

A cylindrical concrete platform with a weight per square metre of $Q\ kN/m^2$ and a diameter of 2 m is carried by a vertical steel strut ($E = 202\ GPa,\ S_y = 300\ MPa$) with a height of 5 m and a diameter of 50 mm as shown in Figure Q2. The bottom end of the strut is rigidly fixed to the ground and the top end of the strut can be assumed to freely support the concrete platform. A Rankine constant of $\frac{1}{7500}$ for pinned ends is to be assumed for the vertical steel strut.

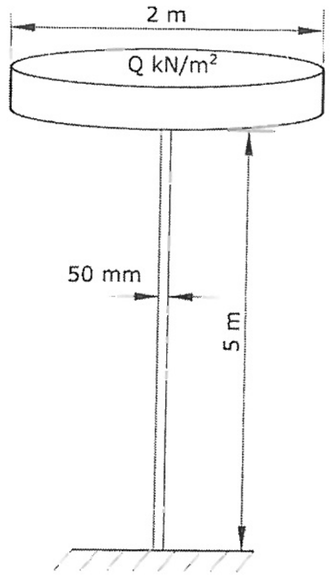


Figure Q2

For a factor of safety of 1.5 calculate:

a) the maximum uniformly distributed concrete weight of $Q\ kN/m^2$ that will just cause the vertical steel strut to buckle, according to Euler buckling theory; and (8)

b) the maximum uniformly distributed concrete weight of $Q\ kN/m^2$ that will just cause the vertical strut to buckle, according to Rankine buckling theory. (8)

[16]

QUESTION 3

The steel shaft (E = 200 GPa) of an electric motor has a diameter of 20 mm, overhangs by 250 mm and is fitted with a pulley of diameter 150 mm at the free end as shown in Figure Q3. The pulley drives a V-belt at a constant speed of 2500 rpm such that the tension on the load size is $T_1 = 300\ N$ and the tension on the slack side is T_2. The V-belt has a tension ratio $\frac{T_1}{T_2} = 2.2$ and a belt angle of approach and departure of 20° at the driving pulley as shown.

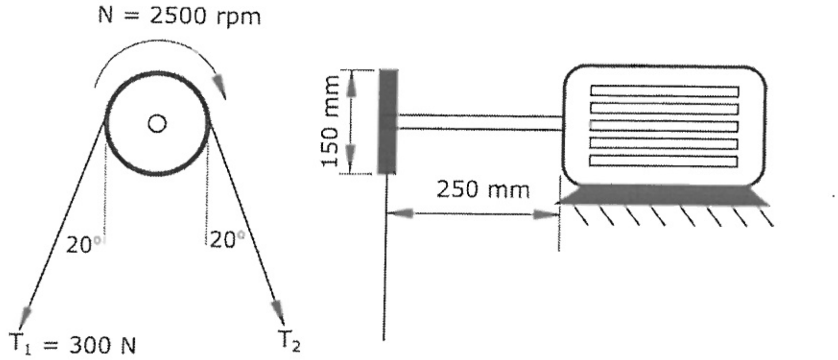


Figure Q3

Calculate:

a) the maximum principal stresses experienced by the overhanging shaft; (12)
b) the angle of orientation of the maximum principal stresses and (2)
c) the maximum shear stress experienced in the shaft of the electric motor. (2)

[16]

QUESTION 4

A solid steel rod (E= $204\ GPa, \nu = 0.28$) of diameter 60 mm just fits inside a 5 mm thick copper tube (E= $100\ GPa, \nu = 0.34$) of the same length. An axial compressive force of 210 kN is applied to the solid steel rod only as shown in Figure Q4.

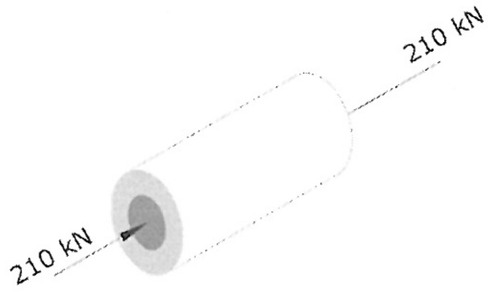


Figure Q4

Calculate:

a) the hoop stress set up in the copper tube; (11)
b) the change in diameter of the solid steel rod and; (3)
c) the percentage change in volume of the solid steel rod. (4)

[18]

QUESTION 5

A cylinder with closed ends has an outer diameter D and a wall thickness $t = 0.1D$ and is subjected to an internal pressure p_i. Determine the percentage error involved in using thin walled cylinder theory instead of thick walled cylinder theory to calculate:

a) the maximum value of hoop stress; and (8)
b) the longitudinal stress in the cylinder. (6)

[14]

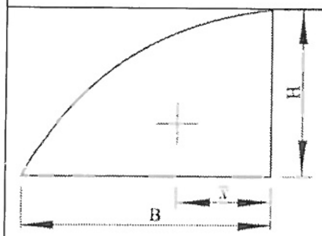
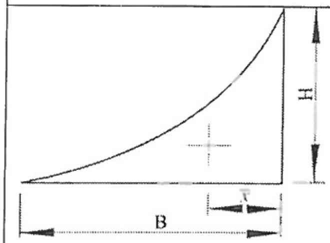
QUESTION 6

A close-ended thick cylinder is to have an internal diameter of 120 mm and a working internal pressure of 60 MPa. Find the needed wall thickness if the factor of safety $n = 2.0$ and the yield stress of the thick cylinder material is 250 MPa using the maximum shear stress theory.

[10]

TOTAL MARKS: 90
FINAL MARKS: 100

ANNEXTURE 1: FORMULA SHEET

1. Quarter elliptical leaf-spring	Maximum bending stress: $\sigma = \frac{6WL}{bnt^2}$ Maximum deflection: $\delta = \frac{6WL^3}{bEnt^3}$
2. Deflection of beams	Deflection of beam at position B relative to A: $$y_{B/A} = \frac{\sum_{i=1}^{n} \int_A^B M(x)_i dx \cdot \bar{x}_i}{\sum_{i=1}^{n} EI_i}$$ Slope of beam at position B relative to A: $$\theta_{B/A} = \frac{\sum_{i=1}^{n} \int_A^B M(x)_i dx}{\sum_{i=1}^{n} EI_i}$$ For $i = 1, 2, \ldots, n$ areas under the bending moment diagram. Where $\bar{x}_i$ is the centroidal distance of area i from position B.
	$\bar{x} = \frac{3}{8}B$ $Area = \frac{2}{3}BH$
	$\bar{x} = \frac{1}{4}B$ $Area = \frac{1}{3}BH$
3. Buckling of Struts	Euler Buckling: $P_E = \frac{\pi^2 EI}{L_e^2}$ Rankine Buckling: $P_R = \frac{S_y A}{\left[1+a\left(\frac{L_e}{K}\right)^2\right]}$ Validity Limit: $\left(\frac{L_e}{K}\right)_{lim} = \sqrt{\frac{2\pi^2 E}{S_y}}$
4. Transformation of Stress	Direct and shear plane stresses on an oblique plane θ degrees (anticlockwise) from the vertical axis: $$\sigma_\theta = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y)Cos2\theta + \tau_{xy}Sin2\theta$$ $$\tau_\theta = \frac{1}{2}(\sigma_x - \sigma_y)Sin2\theta - \tau_{xy}Cos2\theta$$

	Maximum principal direct stresses: $$\sigma_{1,2} = \frac{1}{2}(\sigma_x + \sigma_y) \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$ Maximum principal shear stress: $$\tau_{max} = \pm\frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} = \pm\frac{1}{2}(\sigma_1 - \sigma_2)$$ Direction of maximum principals stresses: $$tan\ 2\theta = \frac{2\tau_{xy}}{(\sigma_x - \sigma_y)}$$
5. Analysis of Strain	Bi-axial strain: $$\varepsilon_x = \frac{\Delta x}{x} = \frac{\sigma_x}{E} - \frac{\upsilon\sigma_y}{E};\ \sigma_x = \frac{E}{(1-\nu^2)}(\varepsilon_x + \nu\varepsilon_y)$$ $$\varepsilon_y = \frac{\Delta y}{y} = \frac{\sigma_y}{E} - \frac{\upsilon\sigma_x}{E};\ \sigma_y = \frac{E}{(1-\nu^2)}(\varepsilon_y + \nu\varepsilon_x)$$ $$\varepsilon_A = \frac{\Delta A}{A} = \varepsilon_y + \varepsilon_x$$ Tri-axial volumetric strain: $$\varepsilon_x = \frac{\Delta x}{x} = \frac{\sigma_x}{E} - \frac{\upsilon\sigma_y}{E} - \frac{\upsilon\sigma_z}{E}$$ $$\varepsilon_y = \frac{\Delta y}{y} = \frac{\sigma_y}{E} - \frac{\upsilon\sigma_x}{E} - \frac{\upsilon\sigma_z}{E}$$ $$\varepsilon_z = \frac{\Delta z}{z} = \frac{\sigma_z}{E} - \frac{\upsilon\sigma_x}{E} - \frac{\upsilon\sigma_y}{E}$$ $$\varepsilon_v = \frac{\Delta V}{V} = \varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{\sigma_x + \sigma_y + \sigma_z}{E}(1-2\nu)$$ Strain in circular shafts: $$\varepsilon_L = \frac{\Delta L}{L} = \frac{1}{E}(\sigma_L - 2\upsilon\sigma_D)$$ $$\varepsilon_D = \frac{\Delta D}{D} = \frac{1}{E}(\sigma_D - \upsilon\sigma_D - \upsilon\sigma_L)$$ $$\varepsilon_v = \frac{\Delta V}{V} = \varepsilon_L + 2\varepsilon_D$$ Strain in thin cylinders: $$\varepsilon_L = \frac{\Delta L}{L} = \frac{\sigma_L}{E} - \frac{\upsilon\sigma_H}{E} = \frac{pd}{4tE}(1-2\upsilon)$$ $$\varepsilon_H = \frac{\Delta H}{H} = \frac{\sigma_H}{E} - \frac{\upsilon\sigma_L}{E} = \frac{pd}{4tE}(2-\upsilon)$$

	$\varepsilon_v = \varepsilon_L + 2\varepsilon_H = \frac{pd}{4tE}(5-4v)$ Strain in thin spheres: $\varepsilon_H = \frac{\Delta H}{H} = \frac{1}{E}(\sigma_H - v\sigma_H) = \frac{pd}{4tE}(1-v)$ $\varepsilon_v = 3\varepsilon_H = \frac{3pd}{4tE}(1-v)$ Elastic constants: $E = 2G(1+ v);\ E = 3K(1-2v)$ Direct and shear plane strains on an oblique plane θ degrees (anticlockwise) from the vertical axis: $\varepsilon_\theta = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)Cos2\theta + \frac{1}{2}\gamma_{xy}Sin2\theta$ $\gamma_\theta = -(\varepsilon_x - \varepsilon_y)Sin2\theta + \gamma_{xy}Cos2\theta$ Maximum principal direct strains: $\varepsilon_{1,2} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) \pm \frac{1}{2}\sqrt{(\varepsilon_x - \varepsilon_y)^2 + \gamma_{xy}^2}$ Direction of maximum principals strains: $tan\ 2\theta = \frac{\gamma_{xy}}{(\varepsilon_x - \varepsilon_y)}$ Shear strain: $\gamma_{xy} = \frac{\tau_{xy}}{G}$
6. Thick Cylinders	Radial stress: $\sigma_r = A - \frac{B}{r^2}$ Hoop stress: $\sigma_h = A + \frac{B}{r^2}$ Stresses in thick cylinders due to an internal pressure P_i and external pressure, P_o: $\sigma_r = \frac{P_i r_i^2}{r_o^2 - r_i^2}\left[1 - \frac{r_o^2}{r^2}\right] - \frac{P_o r_o^2}{r_o^2 - r_i^2}\left[1 - \frac{r_i^2}{r^2}\right]$ $\sigma_h = \frac{P_i r_i^2}{r_o^2 - r_i^2}\left[1 + \frac{r_o^2}{r^2}\right] - \frac{P_o r_o^2}{r_o^2 - r_i^2}\left[1 + \frac{r_i^2}{r^2}\right]$ $\sigma_a = \frac{P_i r_i^2 - P_o r_o^2}{(r_o^2 - r_i^2)}$ Stresses in thick cylinders due to an internal pressure only (P_o= 0):

	$\sigma_r = \frac{P_i r_i^2}{(r_o^2 - r_i^2)}\left[1 - \frac{r_o^2}{r^2}\right]$ $\sigma_h = \frac{P_i r_i^2}{(r_o^2 - r_i^2)}\left[1 + \frac{r_o^2}{r^2}\right]$ $\sigma_a = \frac{P_i r_i^2}{(r_o^2 - r_i^2)}$ Stresses in thick cylinders due to an external pressure only ($P_i = 0$): $\sigma_r = \frac{-P_o r_o^2}{(r_o^2 - r_i^2)}\left[1 - \frac{r_i^2}{r^2}\right]$ $\sigma_h = \frac{-P_o r_o^2}{(r_o^2 - r_i^2)}\left[1 + \frac{r_i^2}{r^2}\right]$ $\sigma_a = \frac{-P_o r_o^2}{(r_o^2 - r_i^2)}$ Diametral shrinkage allowance for compound thick cylinder: $s.a = 2r_{int}\left(\frac{1}{E_O}(\sigma_{h,O,int} + \upsilon_O P_{int}) - \frac{1}{E_I}(\sigma_{h,I,int} + \upsilon_I P_{int})\right)$ Diametral shrinkage allowance for shaft and hub: $s.a = 2r_{int}\left(\frac{1}{E_O}(\sigma_{h,O,int} - \upsilon_O P_{int}) - \frac{1}{E_I}(-P_{int} + \upsilon_I P_{int})\right)$ Torque transmitted by a shrink fit: $T = 2\pi\mu r_{int}^2 L P_{int}$ Frictional force to separate a shrink fit: $F = 2\pi\mu r_{int} L P_{int}$
7. Failure theories	Ductile materials: Failure occurs when: Maximum shear stress (Tresca): $\sigma_1 - \sigma_3 \geq \frac{S_y}{n}$ Maximum shear strain energy (von Mises): $(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \geq 2\left(\frac{S_y}{n}\right)^2$ Brittle materials: Failure occurs when:

	Maximum principal stress (Rankine): $\sigma_1 \geq \frac{S_{ut}}{n} (if\ \sigma_1 > 0)\ or\ \sigma_3 \geq -\frac{S_{ut}}{n} (if\ \sigma_3 < 0)$ Modified Mohr: $Quadrant\ 1: \sigma_1 \geq \frac{S_{ut}}{n}$ $Quadrant\ 2: \frac{\sigma_3}{S_{ut}} - \frac{\sigma_1}{S_{uc}} \geq \frac{1}{n}$ $Quadrant\ 3: \sigma_3 \geq -\frac{S_{uc}}{n}$ $Quadrant\ 4: \frac{\sigma_1}{S_{ut}} - \frac{\sigma_3}{S_{uc}} \geq \frac{1}{n}$