UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

PHYSICS **AUCKLAND PARK KINGSWAY CAMPUS**

PHY0023: ASTROPHYSICS 1
STELLAR STRUCTURE & EVOLUTION

EXAMINATION
JANUARY 2017

INTERNAL EXAMINER: **Prof CA Engelbrecht**

EXTERNAL MODERATOR: Dr I Loubser (NWU)

TIME: 2.5 HOURS **MARKS: 100**

This paper consists of 6 pages, including this cover.

Please read the following instructions carefully:

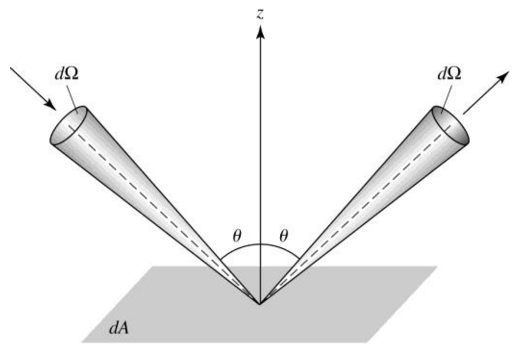
1. **ANSWER ANY FOUR QUESTIONS**
2. **USEFUL INFORMATION APPEARS AT THE END OF THE PAPER**
3. No programmable calculators are allowed.

GENERAL NOTE FOR ALL THE QUESTIONS: EXPLAIN THE LOGICAL SEQUENCE OF ALL THE STEPS IN THE DERIVATIONS; DO NOT JUST WRITE DOWN THE EQUATIONS WITH NO COMMENT ......

QUESTION 1 **[25]**

(a) Use the accompanying figure to derive the following relation for the *radiation pressure* inside a star:

$$P_{rad} = \frac{4\pi}{3c} I_\lambda$$



(9)

(b) Define a 'grey atmosphere' and then derive the following relation between radiative flux and radiation pressure:

$$\frac{dP_{rad}}{dr} = -\frac{\bar{\kappa}\rho}{c} F_{rad} \qquad (9)$$

(c) Use the relation in given in part (b) to derive the following standard equation for the radiative temperature gradient in a star:

$$\frac{dT}{dr} = -\frac{\bar{\kappa}\rho}{4ac}\frac{1}{T^3}\frac{L_r}{4\pi r^2}\ . \qquad (3)$$

(d) Describe what is meant by *limb darkening* in stellar physics. It is recommended that you include an appropriate *diagram* as part of your description. (4)

QUESTION 2 **[25]**

(a) (i): Start with an appropriate expression for the *modification* of the *intensity* of a beam of photons *moving through the plasma* inside a star and *derive* the *Transfer Equation*, in the following form:

$$-\frac{1}{\kappa_\lambda \rho}\frac{dI_\lambda}{ds} = I_\lambda - S_\lambda$$

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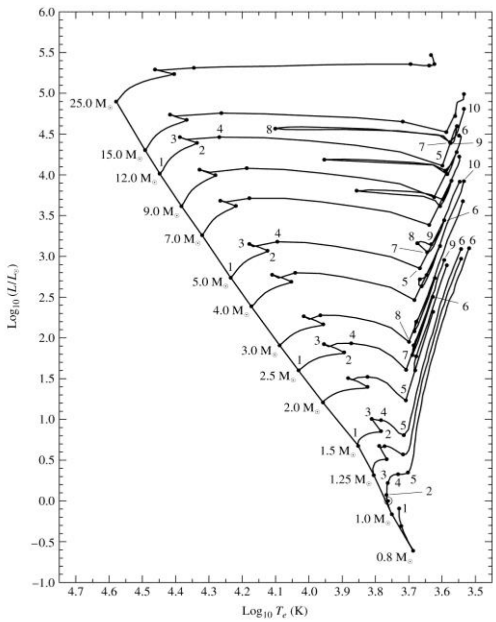
(ii): Explain the *physical* meaning of the quantity expressed as S_λ and describe the physical implications that the formula derived in (i) has for the *equilibrium* state of the radiation field inside a typical star. (6)

(b) Define the *Rosseland mean opacity* and explain its meaning. (3)

(c) Explain why stars (of *all* masses) will *not* evolve past the upward sloping “wall” seen on the low-temperature side of the composite plot of evolutionary tracks shown in the accompanying diagram:



(5)

(d) The Eddington approximation delivers the following result:

$$\langle I \rangle = \frac{3\sigma}{4\pi} T_e^4 \left(\tau_v + \frac{2}{3} \right).$$

Apply this to a star in the blackbody radiation approximation and explain why the optical depth of a star’s photosphere is 2/3. (7)

(e) Briefly describe what happens to a *high-mass* star while it is in the main sequence stage of evolution.

(4)

QUESTION 3 **[25]**

(a) Give a single, *qualitative* explanation for the sharp change in stellar structure that appears to occur somewhere between 1 and 2 solar masses in the following two plots:

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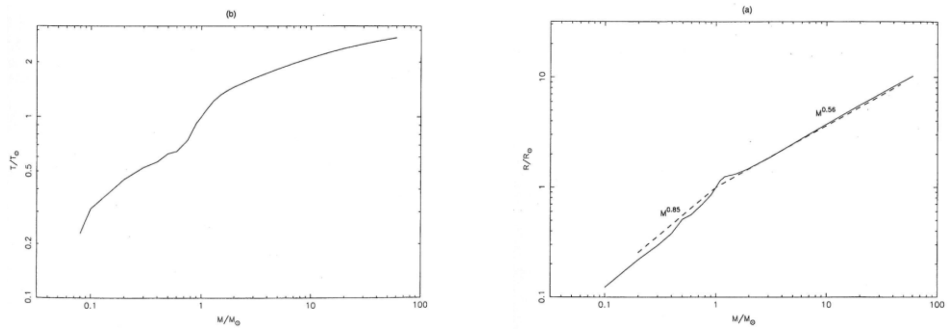


(4)

(b) Use an appropriate diagram to derive the equation for the *adiabatic temperature gradient* in a self-gravitating ideal gas:

$$\frac{dT}{dr}\Big|_{ad} = -\left(1-\frac{1}{\gamma}\right)\frac{\mu m_{\mathrm{H}}}{k}\frac{GM_r}{r^2}$$

(12)

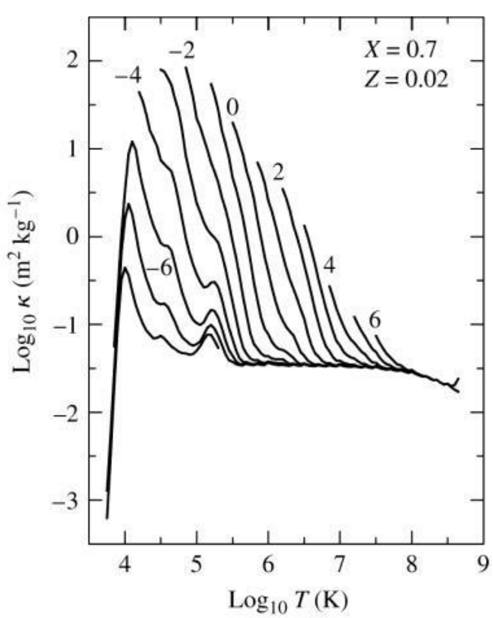
(c) Define the *Schwarzschild* and *Ledoux* criteria (respectively) for convective stability (you may use appropriate formulas from the other questions in this paper to clarify your definitions). Also explain the physical differences between these two criteria. (7)

(d) Explain *which* ion is responsible for the evolution of a *protostar* '*down the Hayashi track*' and *how* that ion fulfils this role.

(2)

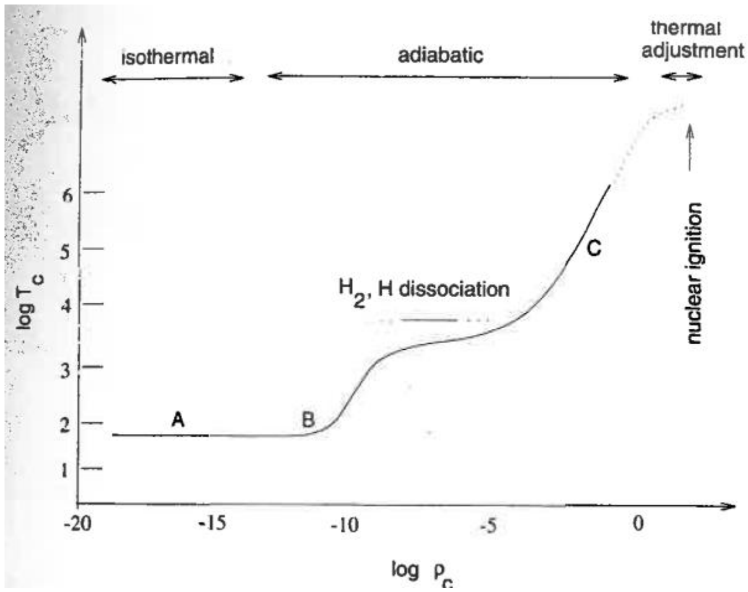
QUESTION 4 **[25]**

(a) Verify the Kramer's opacity law in the accompanying figure.



(4)

(b) Discuss the physics that causes a *protostar* to evolve along the 'track'' running along the solid line from point A to point C on the accompanying diagram:



(7)

(c) Apply the Eddington approximation to the following equation:

$$\frac{dP_{rad}}{d\tau_v} = \frac{1}{c}F_{rad}$$

and derive the result $$\langle I \rangle = \frac{3\sigma}{4\pi}T_e^4\left(\tau_v + \frac{2}{3}\right)$$ (10)

(d) Explain why some stages of stellar evolution proceed quickly while others proceed slowly. (4)

QUESTION 5 **[25]**

(a) Derive the following expression for the *Jeans length* and explain what the *Jeans length* is:

$$R_J = \left(\frac{15kT}{4\pi G\mu m_H \rho_0}\right)^{1/2}$$

(8)

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(b) Explain what is meant by a *partial ionisation zone* (PIZ) in a star and provide some examples of such zones.

(3)

(c) Explain, qualitatively, how the *kappa mechanism* works to enable regular pulsations (oscillations) in a star. (9)

(d) Develop an argument to support the statement that, under conditions of *low density* and *high temperature*, the *maximum* value of the luminosity that still allows *hydrostatic equilibrium* to be maintained in a shell with radius r, is given by:

$$L_{r,\max} = \frac{4\pi c}{\bar{\kappa}} G M_r .$$ (5)

END OF EXAM

THE FOLLOWING EQUATIONS MAY BE USED WITHOUT PROOF:

$$F_\lambda d\lambda = \int I_\lambda d\lambda \cos\theta d\Omega \qquad \langle I_{BB} \rangle \equiv B = \frac{\sigma}{\pi} T^4 \qquad F_{BB} = \frac{L_r}{4\pi r^2}$$

$$P_{rad,\lambda} d\lambda = \frac{1}{c} I_\lambda d\lambda \int_0^{2\pi}\int_0^{\pi} \cos^2\theta (d\Omega)$$

for black body radiation:

$$P_{rad} = \frac{1}{3} a T^4$$

BASIC EQUATIONS OF STELLAR STRUCTURE:

$$\frac{dP}{dr} = -\rho g \qquad \frac{dM_r}{dr} = (4\pi r^2)\rho \qquad \frac{dL_r}{dr} = (4\pi r^2)\rho\varepsilon$$

$$\frac{dT}{dr} = -\frac{3}{4ac}\frac{\bar{\kappa}\rho}{T^3}\frac{L_r}{4\pi r^2}$$