FACULTY OF SCIENCE FAKULTEIT NATUURWETENSKAPPE

| DEPARTMENT OF PHYSICS /DEPARTEMENT FISIKA |  |
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| MODULE: | PHY1A2E |
| CAMPUS | APK |
| SUPPLEMENTARY EXAMINATION |  |

LECTURER
MODERATOR
DURATION 150 min*

DOOMNULL UNWUCHOLA
DR C. J. SHEPPARD
MARKS 132

THIS PAPER CONSISTS OF 9 PAGES INCLUDING THE COVER PAGE
INSTRUCTIONS: Answer ALL questions Use $g=10 \mathrm{~m} / \mathrm{s}^{2}$

## Question 1 [14]

1.1. The period $\boldsymbol{T}$ of a simple pendulum somehow depends on the length $\boldsymbol{I}$ of the string, the acceleration because of gravity $\boldsymbol{g}$ and possibly on the mass $\boldsymbol{m}$ of the bob. Strictly using dimensional analysis, derive the form of equation that relates these quantities.
1.2. The lifting force $\boldsymbol{F}$ on a bird's wing somehow depends on the area $\boldsymbol{A}$ of the wing, the speed vof the bird and the air density $\dot{\boldsymbol{\rho}}$. Make use of dimensional analysis to derive an expression of the lifting in terms of other quantities.
1.3. When a bird glides the lifting force on it is proportional to the square of its speed and the area of its wings. If a 1 kg bird glides horizontally at $5 \mathrm{~m} / \mathrm{s}$, how fast would a 3 kg bird with the same proportions glide horizontally? (When a bird glides horizontally the lifting force is equal to its weight.)
1.4. The mass of gold needed to plate a sphere of mass 0.3 kg to a certain thickness is $10^{-3} \mathrm{~kg}$. Calculate the mass of gold needed to plate a sphere of the same material but mass 2.4 kg to the same thickness as for the 0.3 kg mass.

## Question 2 [24]

2.1 When a force $\boldsymbol{F}$ is applied horizontally, to a mass $\boldsymbol{m}$ at the end of a string as in the figure below, the string makes an angle of $30^{\circ}$ with the vertical.
2.1.1 If the force is increased to $2 F$ calculate be the angle with the vertical.
2.1.2 If $\boldsymbol{F}=20 \mathrm{~N}$ calculate the mass $\boldsymbol{m}$ and the tensions in the string in each case.

2.2 A girl drops a ball from the top of a 20 m high building, and at the same instant a boy throws a ball upwards from the ground with an initial velocity of $20 \mathrm{~m} / \mathrm{s}$. Calculate:
2.2.1 how much time has elapsed when the balls pass each other
2.2.2 how far above the ground they will pass each other
2.2.3 their velocities when they pass each other.
2.3 A projectile is launched with initial velocity $\boldsymbol{u}$ at an angle $\boldsymbol{\theta}$ relative to the horizontal. The launching and landing points lie on the same horizontal plane. If the magnitude of the velocity of the projectile at the maximum height is found to be $\sqrt{\frac{6}{7}}$ times that at half the maximum height, show that the launch angle $\boldsymbol{\theta}$ is $30^{\circ}$.
2.4 A balloon rises from rest on the ground with a constant acceleration a. As it leaves the ground, a gun placed at a horizontal distance $\mathbf{x}$ from the balloon fires a shell with a velocity $u$ at an angle $\boldsymbol{\theta}$ to the horizontal. If the shell hits the balloon show that:

$$
\begin{equation*}
u^{2} \sin 2 \theta=(a+g) x \tag{5}
\end{equation*}
$$

## Question 3 [24]

3.1 A ball on the end of a string, as shown in the figure below, is set in a nonuniform circular motion in a vertical circle of radius $\mathbf{R}$. Prove that the tension in the string at the lowest point exceeds that at the highest point by 6 times the weight of the ball.

3.2 The string of length 4 m , in the figure below, attached to a ball of mass 0.5 kg undergoing a non-uniform circular motion in the vertical plane breaks when the ball is at the lowest point. If the ball then hits the ground at a horizontal distance 8 m and vertical distance 20 m from the position where the string breaks, calculate:
3.2.1 the time $\boldsymbol{t}$ the ball travelled from when the string breaks to when it hits the ground,
3.2.2 the velocity $V_{B}$ of the ball at the moment the string breaks,
3.2.3 the tension of the string just before it breaks.


## Question 4 [21]

4.1. Consider a car travelling around an inclined curve of radius $\boldsymbol{r}$ at a velocity $\boldsymbol{v}$, as shown in the figure below.
4.1.1 If the road is not smooth and banked at $\boldsymbol{\theta}$ to the horizontal, and the minimum coefficient of friction between the tyres of the car and the road, which will prevent the car from skidding sideways is $\mu$. Show that:

$$
\begin{equation*}
\frac{\sin \theta+\mu \cos \theta}{\cos \theta-\mu \sin \theta}=\frac{v^{2}}{r g} \tag{5}
\end{equation*}
$$

Make use of the equation given in 4.1.1:
4.1.2 when $\boldsymbol{\theta}=\mathbf{0}$, give the expression of $\boldsymbol{\mu}$,
4.1.3 when $\boldsymbol{\mu}=\mathbf{0}$, give the expression of $\boldsymbol{\theta}$.

4.2 An object of mass $\boldsymbol{m}_{1}$ hangs from a string that passes over a massless pulley as shown in the figure below. The string connects to a second very light pulley. A second string passes around this pulley with one end attached to a wall and the other to an object of mass $m_{2}$ on a frictionless, horizontal table. With the aid detailed free-body diagrams:
4.2.1 if $\boldsymbol{a}_{\mathbf{1}}$ and $\boldsymbol{a}_{\mathbf{2}}$ are the accelerations of $\boldsymbol{m}_{\boldsymbol{1}}$ and $\boldsymbol{m}_{\boldsymbol{2}}$ respectively, show that

$$
\begin{equation*}
a_{2}=2 a_{1} \tag{3}
\end{equation*}
$$

4.2.2 show how the tensions in the strings are related,
4.2.3 Derive the expressions for the tensions in the strings in terms of $\boldsymbol{m}_{1}, \boldsymbol{m}_{\mathbf{2}}$ and $g$ only.

4.3 A box is on a rough inclined plane making an angle $\boldsymbol{\theta}$ to the horizontal. Just to get the box moving up the plane requires 6 times the force required to just to get it moving down the plane. With the force being applied parallel to the plane in each case, show that the coefficient of friction between the box and the plane can be expressed as:

$$
\begin{equation*}
\boldsymbol{\mu}=\frac{7}{5} \tan \theta . \tag{5}
\end{equation*}
$$

## Question 5 [18]

5.1 A 1.5 kg block at rest on a plane making an angle of $30^{\circ}$ with the horizontal is struck by a 0.02 kg bullet travelling at $300 \mathrm{~m} / \mathrm{s}$ parallel to the plane. The bullet becomes embedded in the block forming a composite mass which travels $\boldsymbol{x}$ meters up the plane, as shown in the figure below. If the coefficient of friction between the block and the plane is 0.2 , calculate $\boldsymbol{x}$.

5.2 The small box shown in the figure below slides a distance $\mathbf{S}_{1}$ down the plane inclined at $\boldsymbol{\theta}_{1}$ to the horizontal. This plane makes a smooth transition to another plane, which makes an angle $\boldsymbol{\theta}_{2}$ with the horizontal. If the kinetic coefficient of friction between the block and the planes is the same, $\boldsymbol{\mu}$, Derive how far the block will slide up the second plane $\mathbf{S}_{\mathbf{2}}$ in terms of $\boldsymbol{S}_{1}, \boldsymbol{\theta}_{1}$, $\boldsymbol{\theta}_{2}$ and $\boldsymbol{\mu}$ only.


## 5.3


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Given that $m_{1}>m_{2}>m_{3}>\boldsymbol{m}_{4}$ in the pulley system, with the aid of detailed free-body diagrams and equations, derive the acceleration $a$ of the system in terms of $\boldsymbol{m}_{1}, \boldsymbol{m}_{2}$, $m_{3}, m_{4}$ and $g$ only.

## Question 6 [31]

6.1 The box of mass $m$ as shown in the diagram slides a distance $\mathbf{S}_{1}$ from rest down the slope making an angle $\boldsymbol{\theta}$ to the horizontal, travels a distance $\mathbf{S}_{\mathbf{2}}$ along a horizontal section at same journey compresses the spring $\mathbf{x}$ distance from its original length before coming to rest. If also the stiffness constant of the spring is $\mathbf{k}$, by the applying concept of law of conservation of work and energy, derive the expression:
6.1.1 for the kinetic coefficient of friction $\mu$ between the box and the surface in terms of $\mathbf{S}_{1}, \mathbf{S}_{\mathbf{2}}, \mathbf{m}, \mathbf{k}, \mathbf{x}$ and $\boldsymbol{\theta}$ given that $\boldsymbol{\mu}$ it the same for the whole trip,
6.1.2 for $\mathbf{m}=3 \mathrm{~kg}, \mathbf{S}_{\mathbf{1}}=4 \mathrm{~m}, \mathbf{S}_{\mathbf{2}}=3 \mathrm{~m}, \mathbf{k}=10^{6} \mathrm{~N} / \mathrm{m}, \mathbf{x}=0.00651 \mathrm{~m}$ and $\theta=30^{\circ}$, calculate $\mu$.


## 6.3



At equilibrium position neither of the springs with stiffness constants $\mathrm{k}_{1}$ and $\mathrm{k}_{2}$ respectively is stretched or compressed. The mass $m$ is displaced $x$ from equilibrium and then released from rest. Derive the expression
6.3.1 for the effective stiffness constant of the system keffi in terms of $\mathrm{k}_{1}$ and $\mathrm{k}_{2}$,
6.3.2 for the acceleration of the system in terms of $x, m, \mathrm{k}_{1}$ and $\mathrm{k}_{2}$,
6.3.3 for the velocity of the amplitude $A, x, m, \mathrm{k}_{1}$ and $\mathrm{k}_{2}$.
6.4 If $m=4.0 \mathrm{~kg}, \mathrm{k}_{1}=20 \mathrm{~N} / \mathrm{m}, \mathrm{k}_{2}=30 \mathrm{~N} / \mathrm{m}$ and the mass is displaced $x=0.05 \mathrm{~m}$ from Equilibrium and then released from rest. Calculate:
6.4.1 its acceleration just after being released,
6.4.2 its velocity as it passes the equilibrium point.

END

