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$\qquad$ STUDENT NUMBER: $\qquad$

FACULTY OF SCIENCE

| DEPARTMENT OF PHYSICS |  |
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| MODULE: | PHYE0A2 |
| CAMPUS | APK |
| EXAM | 09 June 2017 |


|  | Student's <br> Mark | Question's <br> Mark |
| :--- | :--- | :---: |
| Q 1 |  | 15 |
| Q 2 |  | 10 |
| Q 3 |  | 20 |
| Q4 |  | 10 |
| Q 5 |  | 15 |
| Q 6 |  | 20 |
|  |  |  |
| Total |  | 90 |

MODERATOR
Prof. H. Winkler
DURATION 150 min
MARKS 90

THIS PAPER CONSISTS OF 20 PAGES INCLUDING THE COVER PAGE
INSTRUCTIONS: Answer ALL questions IN SPACES PROVIDED NO PENCIL

## Question 1 [14]

1.1) Using the kinematic definitions as your point of departure, derive the below expression governing the displacement of a particle under constant acceleration as a function of time. Indicate all relevant arguments in your derivation

$$
\Delta x=v_{x, i} t+1 / 2\left(a_{x} t^{2}\right)
$$

1.2) Starting from the definition of inertia show how this leads to the conservation of linear momentum in a closed system.
1.3) A person on a bicycle moves along a straight road from his front gate to the shops at a constant speed of $8 \mathrm{~m} / \mathrm{s}$. At the shops he then realises that he forgot his wallet at home and races back to fetch it, travelling at a constant speed of $15 \mathrm{~m} / \mathrm{s}$. What is his average speed for the journey to the shop and back? What is his average velocity for the same journey?
1.4) The $x$-component of the velocity of a car changes from $-2 \mathrm{~m} / \mathrm{s} \hat{x}$ to $+5 \mathrm{~m} / \mathrm{s} \hat{x}$ in 5 seconds, find:
(a) Is the car travelling in the positive or negative $x$ direction?
(b) Does $\Delta v$ point in the positive or negative $x$ direction?
(c) Is the $x$ component of the acceleration positive or negative?
(d) Is the car speeding up, slowing down or both?

## Question 2 [16]

2.1) Show that near the earth's surface the conservation of total mechanical energy holds for an object moved through an arbitrary vertical distance.
2.2) A block of mass $m$ is at rest at the top of an incline (angle $\theta$ measured with respect to the horizontal) shown in the figure below. The surface of the incline is rough with a coefficient of friction $\mu$.
(a) Using conservation of energy find the speed of the block m at the bottom of the incline.

(b) With what speed does the mass hit the ground?
(c) How far away from the end of the incline does the block land (the magnitude of d)?
2.3) Two 1 kg carts are joined together and are initially moving to the right at $2 \mathrm{~m} / \mathrm{s}$ on a horizontal frictionless track. The join contains explosives which can be detonated remotely. The explosion creates 18 J of energy, half of which is transformed into internal energy (noise, heat etc.). The remaining energy pushes the carts apart. What are the velocities of the two carts after the explosion?

## QUESTION 3 [23]

3.1) Starting with the idea that a change in an object's momentum with respect to time is caused by the vector sum of all forces acting on the object, derive:

$$
\sum \boldsymbol{F}=m \boldsymbol{a}
$$

3.2) Three masses are connected as shown in the figure below. The direction of the acceleration of mass 3 is shown. The rope connecting the masses is inelastic and the pulleys have no mass and are frictionless. Mass 2 is on a rough surface with a coefficient of friction $\mu$. When the spring connected to mass 1 has been stretched a distance $d$ from its equilibrium position (not stretched), what is the acceleration of the system?

3.3) Show that the work done by a non-dissipative, spatially dependent force is given by:

$$
W=\int_{x i}^{x f} F_{x} d x
$$

3.4) Use the equation in (3.3) to show that the work done by a spring is:

$$
W=\frac{1}{2} k\left(x_{f}^{2}-x_{i}^{2}\right)
$$

3.5) Show that the maximum height reached by a projectile having an initial velocity $\boldsymbol{v}_{i}$ launched at an angle $\theta$ with respect to the horizontal, is given by:

$$
h=\frac{v_{i}^{2}(\sin \theta)^{2}}{2 g}
$$

3.6) A disk $P$ (inertia $=0.4 \mathrm{~kg}$ ) moves with an unknown velocity along a smooth horizontal surface and collides with a disk Q (inertia $=0.7 \mathrm{~kg}$ ) which is at rest. After the collision the two slightly deformed disks move apart (without spinning) with the following velocities: $\mathrm{P}: v_{f}=1.4 \cos \left(20^{\circ}\right) \hat{x}+1.4 \sin \left(20^{\circ}\right) \hat{y}$ and $\mathrm{Q}: v_{f}=0.96 \cos \left(50^{\circ}\right) \hat{x}-0.96 \sin \left(50^{\circ}\right) \hat{y}$.
(a) What is the inertial velocity of disk P?

## QUESTION 4 [23]

4.1) Derive the expression for centripetal acceleration (i.e. $a_{c}=v^{2} / r$ ), stating clearly the logic used.
4.2) Derive an expression for the torque applied to a point-like object, with respect to the particle's rotational acceleration. Show all appropriate figures.
4.3) The figure below represents a ladder (length $\ell$ ) which is leaning on a wall at an angle $\theta$ with respect to the floor. The vertical arrow located at a distance $\ell / 4$ from the top of the ladder represents the location of a man, whose inertia is 80 kg . If the floor and the wall have different coefficients of friction ( $\mu_{1}$ and $\mu_{2}$ ), what is the maximum angle $\theta$ at which the ladder will not slip?

4.4) The tub of a washing machine undergoes a spin cycle, starting from rest and gaining angular speed at a constant rate for 8 s . After these 8 s the angular speed is $5 \mathrm{rev} / \mathrm{s}$. Load-shedding occurs at this point and the tub smoothly slows down to rest in 12 s . How many revolutions does the tub go through since starting?
4.4) A 70 kg woman is standing on the edge of a stationary circular turntable whose radius is 3 m . The turntable has an inertia of $400 \mathrm{~kg} \cdot \mathrm{~m}^{2}$. If the woman walks at a linear speed of $3 \mathrm{~m} / \mathrm{s}$ in a clockwise direction, find the angular speed of the turntable.
5.1) Give the general solution for the displacement of a particle undergoing simple harmonic motion and derive the expressions for its velocity and acceleration.
5.2) A travelling wave is described by $f_{1}(x, t)=A \sin (k x-\omega t)$ when moving to the right. If a wave with the same wavelength and frequency is approaching from the left then find:
(a) The equation of this wave travelling to the left
(b) Show how this combination produces standing waves and find the condition for the location of the nodes. You may use the following:

$$
\sin \alpha+\sin \beta=2 \sin \left(\frac{\alpha+\beta}{2}\right) \cos \left(\frac{\alpha-\beta}{2}\right)
$$

5.4) (a) For a non-compressible fluid with a laminar flow, derive the expression for the conservation of matter, the so called "continuity equation".
(b) Derive Bernoulli's equation including a sketch, stating the logic behind each and every step.

## Question 6 [18]

6.1) (a) What is the period of a simple pendulum on the moon if the length of the string is 0.5 m ? Given: moon mass $=7.3 \times 10^{22} \mathrm{~kg}$ and moon radius $=1.74 \times 10^{6} \mathrm{~m}$
(b) How does this compare to the period of this pendulum on the earth?
6.2) A geosynchronous orbit implies that a satellite remains at a fixed point above the earth's surface at all times as the earth rotates. Show that the height required to achieve this is

$$
\left(r_{e}+h\right)^{3}=G m_{e}\left(\frac{T}{2 \pi}\right)^{2}
$$

6.2) A cord runs over a massless pulley which is 2.3 m away from the wall. One end of the cord is attached to a wall, while the other end is connected to a mass, as in the figure below. The cord has a mass per unit length $\mu=1.3 \mathrm{~g} / \mathrm{m}$. What is the mass of the block if the third harmonic of the cord (standing wave) vibrates with a frequency of 550 Hz ?

6.3) A pipe with an inner diameter of 15 mm carries water with a flow rate of $750 \mathrm{~mm} / \mathrm{s}$. The inner diameter then narrows from 15 mm to 10 mm . Ignore the viscosity of the water. Find the pressure difference between the 10 and 15 mm diameter pipes.
6.4) Two speakers simultaneously emit a note of wavelength $\lambda$. The speakers are separated by a distance of $10 \lambda / 3 \mathrm{~m}$. If a microphone is moved along the line between the speakers' centres, at what positions will the microphone detect a maximum and minimum in the sound intensity?
[4]

