UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

MODULE PHY0017

CAMPUS APK

EXAM JUNE 2016

DATE: 14/06/2016	**SESSION: 08:30**
ASSESSOR(S):	**DR BP DOYLE**
EXTERNAL MODERATOR:	**PROF R DE MELLO KOCH (WITS)**
DURATION: 3 HOURS	**MARKS: 80**

NUMBER OF PAGES: 4 PAGES INCLUDING THIS ONE AND THE INFORMATION SHEET

INSTRUCTIONS: **Answer all the questions.**

Explanations are necessary!

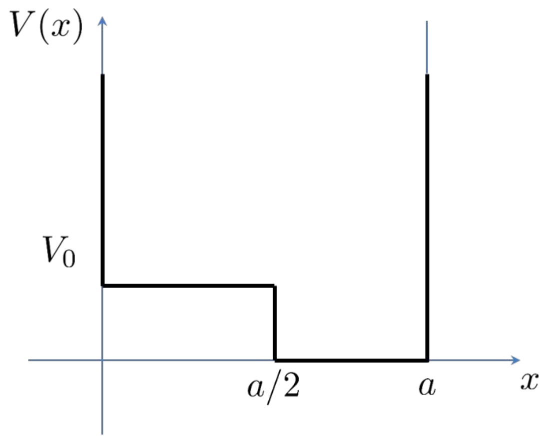
Question 1 [22]

(1a) Derive the first-order correction to the **wave function** for a perturbation, H', in non-degenerate perturbation theory. [7]

(1b) The unperturbed wave functions for the infinite square well are:

$$\psi_n^0(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x\right).$$

If we perturb the system by raising half of the "floor" by a constant amount V_0 as shown in the figure, find the first order correction to the energy. [3]



(1c) The spin-orbit Hamiltonian is given by,

$$H = \left(\frac{e^2}{8\pi\epsilon_0}\right)\frac{1}{m^2c^2r^3}\mathbf{S}\cdot\mathbf{L}$$

Find the first order correction to the energies of the hydrogen atom due to this perturbation. [5]

(1d) The Zeeman perturbation to the Hamiltonian is,

$$H_Z' = -\frac{e}{2m}(\mathbf{L} + 2\mathbf{S})\cdot\mathbf{B}_{\text{ext}}$$

For the weak-field Zeeman effect find the first order correction to the energies of the hydrogen atom. [7]

Question 2 [10]

(2a) Derive Ehrenfests theorem for a time independent operator. [4]

(2b) Use this result to derive the quantum mechanical version of the classical equation $\dfrac{\partial\mathscr{H}}{\partial p_i} = \dot{q}_i$, where q_i is a position coordinate. [6]

Question 3 [13]

(3a) For the time independent Schrödinger equation,

$$H\left|E\right\rangle = E\left|E\right\rangle$$

show that the propagator is given by:

$$U(t) = \sum_E \left|E\right\rangle\left\langle E\right| e^{-iEt/\hbar} \qquad [6]$$

(3b)

(i) By ignoring all paths except the classical one and its neighbours, calculate the propagator, $U(t)$, for a free particle. [6]

(ii) Explain clearly why the method employed in part (i) of this question to determine $A(t)$ cannot normally be used. [1]

Question 4 [21]

(4a) What is the difference conceptually between the active transformation picture and the passive transformation picture? [2]

(4b) This question is in the **active transformation** picture.

(i) Determine the translation operator $T(\varepsilon)$ to first order in ε. [6]

(ii) Now find the conservation law that results from translational invariance in this picture. [7]

(4c) Show that, if the Hamiltonian, H, is parity invariant; then the parity operator, Π, and H commute. [3]

(4d) Describe how the β-decay of ^{60}Co is not invariant under parity. You should probably also draw a picture. [3]

Question 5 [14]

(5a) The action of the rotation operator on position eigenkets is defined as,

$$U[R]\left|x, y\right\rangle = \left|x\cos\phi_0 - y\sin\phi_0, x\sin\phi_0 + y\cos\phi_0\right\rangle$$

(i) To first order, prove that,

$$\langle x, y| I - \frac{i\varepsilon_z L_z}{\hbar}|\psi\rangle = \psi(x + y\varepsilon_z, y - x\varepsilon_z) \qquad [6]$$

(ii) Now show that this leads to,

$$L_z = XP_y - YP_x \qquad [4]$$

(5b) Write down the transformation equation for an infinitesimal rotation of X in the passive picture. Use this to show that,

$$[Y, L_z] = i\hbar X \qquad [4]$$

END of QUESTIONS

INFORMATION SHEET

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V\Psi$$

$$[r_i, p_j] = -[p_i, r_j] = i\hbar\delta_{ij} \quad , \quad [r_i, r_j] = [p_i, p_j] = 0$$

$$[L_x, L_y] = i\hbar L_z \quad , \quad [S_x, S_y] = i\hbar S_z$$

For operators: $[\Omega, \Lambda\theta] = \Lambda[\Omega, \theta] + [\Omega, \Lambda]\theta$ and $[\Lambda\Omega, \theta] = \Lambda[\Omega, \theta] + [\Lambda, \theta]\Omega$

Ehrenfest's theorem: $\frac{d}{dt}\langle\Omega\rangle = \left(\frac{-i}{\hbar}\right)\langle[\Omega, H]\rangle$

$$\int \sin^2(ax)\, dx = \frac{x}{2} - \frac{\sin(2ax)}{4a}$$

$$\int \sin(mx)\sin(nx)\, dx = \left(\frac{\sin[(m-n)x]}{2(m-n)} - \frac{\sin[(m+n)x]}{2(m+n)}\right)$$

$$\hbar = 1.05457 \times 10^{-34} J \cdot s$$

Wavefunctions of the infinite square well: $\psi_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x\right)$

$$a = \frac{4\pi\epsilon_0\hbar^2}{m_e e^2} = \frac{\hbar}{\alpha m_e c} = 5.29177 \times 10^{-11} m$$

$$m_e = 9.10938 \times 10^{-31} kg$$

$$m_p = 1.67262 \times 10^{-27} kg$$

$$\delta(x - x') \equiv \lim_{\Delta\to 0}\frac{1}{(\pi\Delta^2)^{1/2}}\exp\left[\frac{(x-x')^2}{\Delta^2}\right]$$

Fresnel integrals: $\int_0^\infty \cos(t^2)dt = \int_0^\infty \sin(t^2)dt = \sqrt{\frac{\pi}{8}}$

Gaussian integral: $\int_{-\infty}^{\infty} x^2 e^{-\alpha x^2}\, dx = \frac{1}{2\alpha}\left(\frac{\pi}{\alpha}\right)^{1/2}$

$$e^{-ax} = \lim_{N\to\infty}\left(1 - \frac{ax}{N}\right)^N$$

Taylor series: $f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f^{(3)}(a)}{3!}(x-a)^3 + \cdots = \sum_{n=0}^{\infty}\frac{f^{(n)}(a)}{n!}(x-a)^n$

Taylor series for e^x: $1 + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots = \sum_{n=0}^{\infty}\frac{x^n}{n!}$

$$\int \mathscr{D}[x(t)] = \lim_{\substack{\varepsilon\to 0\\ N\to\infty}}\frac{1}{B}\int_{-\infty}^{\infty}\int\int\cdots\int_{-\infty}^{\infty}\frac{dx_1}{B}\cdot\frac{dx_2}{B}\cdots\frac{dx_{N-1}}{B} \quad \text{where} \quad B = \left(\frac{2\pi\hbar\varepsilon i}{m}\right)^{1/2}$$

$$\int_a^b \langle x|x'\rangle\langle x'|f\rangle\, dx' = \langle x|I|f\rangle = \langle x|f\rangle$$

For the hydrogen atom: $\left\langle\frac{1}{r}\right\rangle = \frac{1}{n^2 a}$; $\left\langle\frac{1}{r^2}\right\rangle = \frac{1}{(l^2 + 1/2)n^3 a^2}$; $\left\langle\frac{1}{r^3}\right\rangle = \frac{1}{l(l+1/2)(l+1)n^3 a^3}$

Bohr energies of hydrogen: $E_n = -\left[\frac{m}{2\hbar^2}\left(\frac{e^2}{4\pi\epsilon_0}\right)^2\right]\frac{1}{n^2} = \frac{E_1}{n^2}$