UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS	
MODULE	MATE0A1 CALCULUS OF ONE-VARIABLE FUNCTIONS FOR ENGINEERS
CAMPUS	APK
EXAM	JUNE EXAM 2016

DATE 31/05/2016 — SESSION 12:30 - 14:30

ASSESSOR(S) — MR F CILLIERS, MR A SWARTZ

INTERNAL MODERATOR — DR J MBA

DURATION 2 HOURS — MARKS 70

SURNAME AND INITIALS ______________________________

STUDENT NUMBER ______________________________

CONTACT NUMBER ______________________________

NUMBER OF PAGES: 1 + 12 PAGES

INSTRUCTIONS:
1. ANSWER ALL THE QUESTIONS ON THE PAPER IN PEN.
2. NO CALCULATORS ARE ALLOWED.
3. SHOW ALL CALCULATIONS AND MOTIVATE ALL ANSWERS.
4. IF YOU REQUIRE EXTRA SPACE, CONTINUE ON THE ADJACENT BLANK PAGE AND INDICATE THIS CLEARLY.

Question 1 [8 marks]

For questions 1.1 - 1.8, choose **one** correct answer, and make a cross (X) in the correct block.

Question	**a**	**b**	**c**	**d**	**e**
1.1					
1.2					
1.3					
1.4					
1.5					
1.6					
1.7					
1.8					

1.1 Which one of the following is a negation of [1]

"Tim is inside and Leo is at the pool."

a) Tim is inside or Leo is not at the pool.

b) Tim is inside or Leo is at the pool.

c) Tim is not inside or Leo is at the pool.

d) Tim is not inside and Leo is not at the pool.

e) Tim is not inside or Leo is not at the pool.

1.2 What is the correct translation of the sentence into First Order language : [1]

"The square of x is greater than x whenever x is any positive real number."

a) $\exists x \in \mathbb{R}((x > 0) \wedge (x^2 > x))$

b) $\forall x \in \mathbb{R}((x > 0) \to (x^2 > x))$

c) $\forall x \in \mathbb{R}((x > 0) \wedge (x^2 > x))$

d) $\exists x \in \mathbb{R}((x \geq 0) \to (x^2 \geq x))$

e) None of the above

1.3 If a function $g(x)$ is shifted 1 unit to the right, then shifted 4 units down, and then reflected about the x-axis, the resulting equation is: [1]

a) $g(x+1)-4$

b) $-g(x-1)-4$

c) $-g(x-1)+4$

d) $g(x+1)+4$

e) None of the above

1.4 If $3 \leqslant f(x) \leqslant 7$ for $-1 \leqslant x \leqslant 1$, then which of the following is guaranteed to be true: [1]

a) $-3 \leqslant \int_{-1}^{1} f(x)\,dx \leqslant 7$

b) $-6 \leqslant \int_{-1}^{1} f(x)\,dx \leqslant 14$

c) $3 \leqslant \int_{-1}^{1} f(x)\,dx \leqslant -7$

d) $6 \leqslant \int_{-1}^{1} f(x)\,dx \leqslant 14$

e) None of the above

1.5 Which pair correctly fills the blanks in the following theorem: [1]

If f is continuous on ____ then $\int_a^b f(x)\,dx = F(b) - F(a)$ where F is a function such that ____.

a) (a, b), $F' = f$

b) $[a, b]$, $f' = F$

c) $[a, b]$, $F' = f$

d) (a, b), $f' = F$

e) None of the above

1.6 If $y = (\ln x)^3$, then $\dfrac{dy}{dx} =$ [1]

a) $\frac{3}{x}(\ln x)^2$

b) $3(\ln x)^2$

c) $3x(\ln x)^2 + (\ln x)^3$

d) $3(\ln x + 1)$

e) None of the above

1.7 If $F(x) = x \sin x$, then $F'(\frac{3\pi}{2}) =$ [1]

a) 0

b) 1

c) -1

d) $-\frac{3\pi}{2}$

e) None of the above

1.8 All the functions below, except one, has the property that $f(x)$ is equal to its fourth derivative. Which one of the following does not have this property ? [1]

a) $f(x) = \sin x$

b) $f(x) = e^{2x}$

c) $f(x) = \cos x$

d) $f(x) = e^{-x}$

e) None of the above

Question 2 [7 marks]

a) Fill in the blank spaces in the following table. [3]

p	q	$\neg p$	$\neg q$	$\neg q \to p$	$p \wedge \neg p$
T	T				
T	F				
F	T				
F	F				

b) Answer the following **true/false** questions. For each item (i - iv) make a cross (X) in the corresponding block.

Question	**TRUE**	**FALSE**
i		
ii		
iii		
iv		

i) $\neg p \to q$ is the negation of $p \to q$. [1]

ii) $p \wedge \neg p$ is a contradiction. [1]

iii) $\neg q \to \neg p$ is the converse of $p \to q$. [1]

iv) $\neg q \to p$ is a tautology. [1]

Question 3 [6 marks]

a) Use proof by contrapositive to show that if $\sin\left(\frac{k\pi}{2}\right) \neq -1$, then $k \neq 7$. [2]

b) Solve the inequality and express your answer in interval notation:

$$\frac{2x}{2x-1} \leq 1$$

[2]

c) Prove the identity $\tan x + \dfrac{\cos x}{\sin x - 1} = -\sec x$ [2]

Question 4 [8 marks]

a) Determine the domain of the following function, and express your answer in interval notation:

$$f(x) = \sqrt{x-1} - \sqrt{2-x}$$

[2]

b) Determine the range of the function $g(x) = 3 + \sin(x)$, and express your answer in interval notation: [1]

c) Let:

$$f(x) = \begin{cases} \cos x & \text{if } x < 0 \\ \sqrt{x} & \text{if } x \geq 0 \end{cases}$$

d) Sketch the graph of f. [2]

e) Calculate $\lim_{x\to 0} f(x)$. [2]

f) Is f continuous at $x = 0$? Explain your answer. [1]

Question 5 [5 marks]

Determine the following limits if they exist. Do not use L'Hospital's Rule.

a) $\lim_{x\to 3} \dfrac{\sqrt{x+6}-x}{x-3}$ [3]

b) $\lim_{x\to\infty} \dfrac{2x^2+x-9}{x^2-x+3}$ [2]

Question 6 [5 marks]

a) Suppose $g(x) = \displaystyle\int_2^{\sin x} e^t \, dt$. Use the Fundamental Theorem of Calculus (Part 1) to find $g'(x)$. [2]

b) Use the information provided to find $f(t)$: $f''(t) = \cos t,\ f'(0) = 3,\ f(0) = 5.$ [3]

Question 7 [7 marks]

Evaluate the following integrals:

a) $\displaystyle\int_{-1}^{2} (3x^2 + 3x + 3)\, dx$ [3]

b) $\displaystyle\int \left(\frac{5^x}{2} + \frac{3}{1+x^2}\right) dx$ [2]

c) $\displaystyle\int xe^{x^2}\,dx$ [2]

Question 8 [7 marks]

a) Find $\dfrac{d}{dx}\left(\log_3\left(\cot x + 2^{3x^2}\right)\right)$ [2]

b) Find the slope of the tangent line to the curve at the given point: [3]

$$(x^2+y^2)^2 = 4x^2y \qquad (1,1)$$

c) Prove that $\frac{d}{dx}\left(\csc^{-1} x\right) = \frac{-1}{x\sqrt{x^2-1}}$ [2]

Question 9 [3 marks]

Differentiate the following function **without** using the product or the quotient rule. [3]

$$y = \frac{(e^{2x}+6)^7.\sqrt{x+4}}{(e^{-x}+e^x)^5}$$

Question 10 [4 marks]

Use mathematical induction to prove that for all $n \geqslant 1$ that:

$$\sum_{j=1}^{n} \frac{1}{(2j-1)(2j+1)} = \frac{n}{2n+1}$$

[4]

Question 11 [4 marks]

If f and g are both differentiable and $g(x) \neq 0$, prove the Quotient Rule of differentiation, that is

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

[4]

Question 12 [6 marks]

a) Evaluate the following limit. Use L'Hospital's Rule if necessary: $\lim_{x \to 0} (\cos 3x)^{\frac{5}{x}}$ [3]

b) If $y = \ln(\sec x + \tan x)$, use the definition of the hyperbolic function to show that $\cosh y = \sec x$. [3]