FACULTY OF SCIENCE
UNIVERSITY OF JOHANNESBURG

DEPARTMENT OF PURE AND APPLIED MATHEMATICS

MODULE	**MAT2B10** MULTIVARIABLE AND VECTOR CALCULUS
CAMPUS	**APK**
EXAM	**NOVEMBER 2016**

EXAMINER(S)	MRS C DUNCAN
INTERNAL MODERATOR	DR F SCHULZ
DURATION	2 HOURS
MARKS	50

SURNAME AND INITIALS ______________________________

STUDENT NUMBER ______________________________

CONTACT NUMBER ______________________________

NUMBER OF PAGES: 1 + 12

INSTRUCTIONS:
1. ANSWER ALL QUESTIONS ON THE PAPER IN PEN
2. CALCULATORS ARE ALLOWED
3. INDICATE **CLEARLY** ANY ADDITIONAL PAGES USED

Question 1 [8]

For questions (1.1) - (1.8), please circle only **ONE** correct answer:

(1.1) Find the following limit, if it exists:

$$\lim_{(x,y,z)\to(0,0,0)} \frac{8xy+|z|}{\sqrt{x^2+y^2+z^2}}$$

(a) 0

(b) $\frac{1}{\sqrt{2}}$

(c) 8

(d) $\frac{8}{\sqrt{2}}$

(e) The limit does not exist.

(1.2) If $D_{\mathbf{u}}f(0,0) = c$, for any unit vector $\mathbf{u}$, then $c = 0$.

(a) True

(b) False

(1.3) Find all the saddle points of the function $f(x, y) = x \sin \frac{y}{3}$.

(a) $(0, 3\pi n)$

(b) $(0, \frac{\pi n}{3})$

(c) $(3\pi n, 1)$

(d) $(\frac{3n}{\pi}, 0)$

(e) $(3\pi n, 0)$

(1.4) Find the volume of the solid that lies inside the cylinder $x^2 + y^2 = 9$ and the ellipsoid $2x^2 + 2y^2 + z^2 = 36$.

(a) 260.31

(b) 301.74

(c) 261.29

(d) 292.45

(e) 284.22

(1.5) Find the Jacobian of the transformation $x = 5\alpha \sin \beta$ and $y = 4\alpha \cos \beta$.

(a) 9α

(b) $-20\alpha \sin \beta \cos \beta$

(c) -20α

(d) $-\alpha$

(e) 36α

(1.6) Use Green's Theorem to evaluate $\int_C x^2 y \, dx - xy^2 \, dy$, where C is the circle $x^2 + y^2 = 4$ with counterclockwise orientation.

(a) -8π

(b) -4π

(c) 6π

(d) 2π

(e) none of these

(1.7) Given the position vector $\mathbf{r} = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$, with $|\mathbf{r}| = r$. Then, $\nabla r = \dfrac{2\mathbf{r}}{r}$.

(a) True

(b) False

(1.8) The vector field $\mathbf{F} = xyz\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$ can always be written as the curl of another vector field.

(a) True

(b) False

Question 2 [6]

(2.1) Define clearly what is meant by saying that a function $z = f(x, y)$ is continuous at a point (a, b). (2)

(2.2) Hence, determine the set of points at which the following function is continuous:

$$f(x,y) = \begin{cases} \dfrac{\sin(x-y)}{|x|+|y|} & \text{if } |x|+|y| \neq 0 \\ 0 & \text{if } |x|+|y| = 0 \end{cases}$$

(4)

Question 3 [3]

The directional derivative of $f(x, y)$ at the point $P = (0, 4)$ in the direction of the origin is -2. If $\nabla f(0, 4) = \langle k, k \rangle$ for some $k \in \mathbb{R}$, what is the directional derivative at P in the direction of $\theta = \pi/3$?

Question 4 [4]

Find the dimensions of the rectangle with maximum perimeter that can be inscribed with sides parallel to the coordinate axes in the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$.

Question 5 [6]

The volume of the solid E is given by the following iterated integral

$$V(E) = \int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{\sqrt{16-x^2-y^2}} dzdydx$$

(5.1) Sketch the solid E. (2)

(5.2) Convert the given triple integral to spherical coordinates. You do not need to evaluate the integral. (4)

Question 6 [8]

Consider the double integral

$$\iint\limits_{\Omega} (x+y)\,dxdy$$

with Ω the region bounded by $x = y$, $x = y + \pi$, $x = -2y$ and $x = -2y + \frac{\pi}{2}$.

(6.1) Sketch Ω. (2)

(6.2) Determine the transformation $T(u,v) = (x,y)$ such that $x = g(u,v)$ and $y = h(u,v)$, and calculate the associated Jacobian. (3)

(6.3) Using (6.1) and (6.2), solve the integral $\displaystyle\iint_{\Omega} (x + y)\, dxdy$. (3)

Question 7 [3]

Given a vector field $\mathbf{F} = P\,\mathbf{i} + Q\,\mathbf{j} + R\,\mathbf{k}$. Show that

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_C P\,dx + Q\,dy + R\,dz$$

along a smooth curve C.

Question 8 [5]

The force exerted by an electric charge at the origin on a charged particle at a point (x, y, z) with position vector $\mathbf{r} = \langle x, y, z \rangle$ is $\mathbf{F} = K\mathbf{r}/|\mathbf{r}|^{\mathbf{3}}$, where K is a constant. Find the work done by this force as the particle moves along a straight line from $(2, 0, 0)$ to $(2, 1, 5)$.

Question 9 [4]

Prove that $\int_C \mathbf{F} \cdot d\mathbf{r}$ is independent of path in D if and only if $\int_C \mathbf{F} \cdot d\mathbf{r} = 0$ for every closed path C in D.

Question 10 [3]

If $\mathbf{F} = P\,\mathbf{i} + Q\,\mathbf{j} + R\,\mathbf{k}$ is a vector field on $\mathbb{R}^3$ and P, Q, and R have continuous second-order partial derivatives, then show that

$$\text{div curl } \mathbf{F} = 0.$$