UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

UNIVERSITEIT VAN JOHANNESBURG

DEPARTMENT OF PURE AND APPLIED MATHEMATICS

MODULE **MAT1C3E** (CALCULUS SECTION)
BIO & ENVIRO MATHS AND STATS

CAMPUS **APK**

SUPPLEMENTARY **JULY 2015**

DATE: JULY 2016 **SESSION:**

ASSESSOR: MR. T. MOHUBEDU

INTERNAL MODERATOR: MR. V. VAN APPEL

DURATION: 60 MINUTES **MARKS:** 40

SURNAME AND INITIALS: _____

STUDENT NUMBER: _____

CONTACT NUMBER: _____

Please read the following instructions carefully

1. Answer all questions on the paper in pen.
2. This paper consists of 8 pages including the cover page.
3. **Show all calculations.**
4. **Calculators are allowed.**

1. Given $f(x) = \ln 2 - \cos x$

1.1 Find the first derivative of f. [1]

1.2 Find the second derivative of f. [1]

2. Find the first derivative: $f(x) = \ln(\ln(3x + e)$ [3]

3 Use the product rule for derivatives to find the derivative: [2]

$f(x) = (3 - x^2)(1 - 4x)$

4 Use the product rule for derivatives to find the derivative: [3]

$$f(x) = \frac{\sin x}{1 + 2e^x}$$

5. Given $x^2 + 4xy + y^4 = 1$

5.1 Use implicit differentiation to find y'. [3]

5.2 Find the slope of the tangent line(s) to f at $x = 0$. [3]

6. An object is tossed upward at 10 m/s from a height of 100 m. The distance above the ground is given by $M(t) = 100 + 10t - 4.9t^2$.

6.1 Find the time when the object reaches a critical point. [2]

6.2 Find the maximum height of the object. [1]

6.3 Find the time it takes the object to hit the ground. [2]

7. Suppose a population of bacteria grows according to $P(t) = 10e^t$, and the mass per individual by $m(t) = 1 - t$ for $t \geq 0$.

7.1 Find the equation for the total mass of the population. [1]

7.2 Find the derivative of the total mass. [2]

7.3 Is the total mass of the population increasing or decreasing? Motivate your answer. [2]

7.4 When will the total mass reach zero? [2]

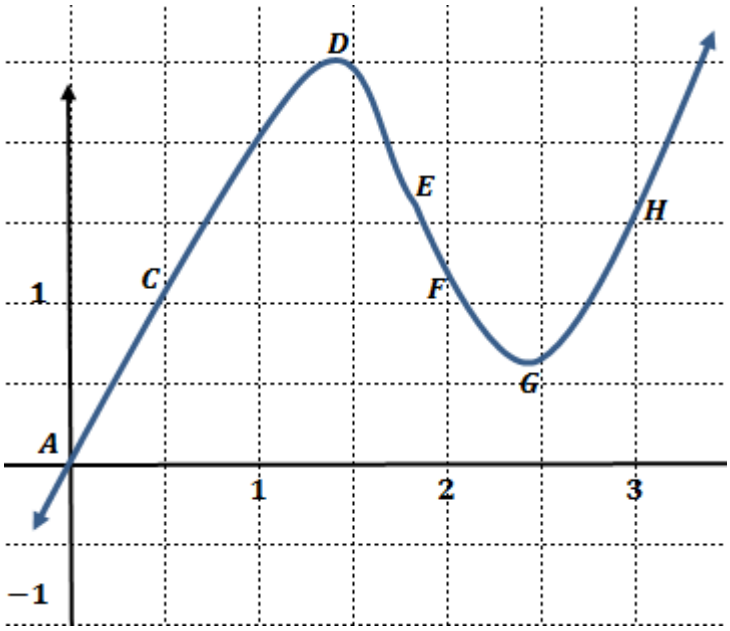
8. Which point on the given graph is:

8.1 a critical point. [1]

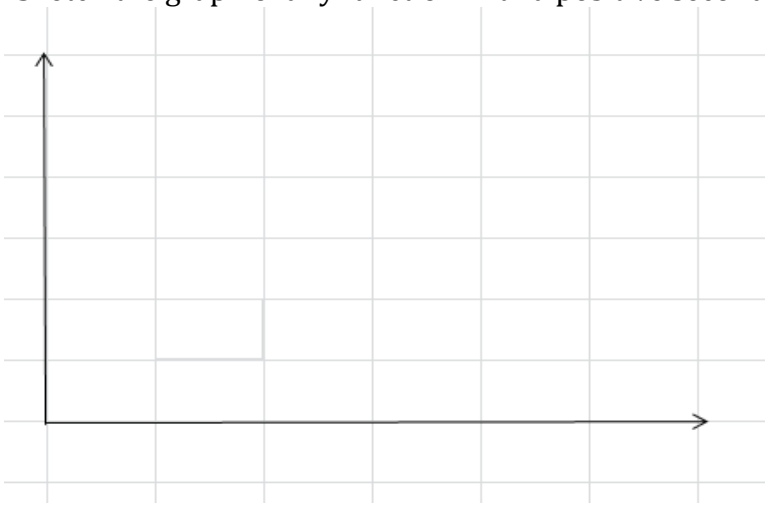
8.2 a point of inflection. [1]

8.3 a point with a positive derivative [1]

8.4 a point with a negative second derivative. [1]



9. Sketch the graph of any function with a positive second derivative. [1]



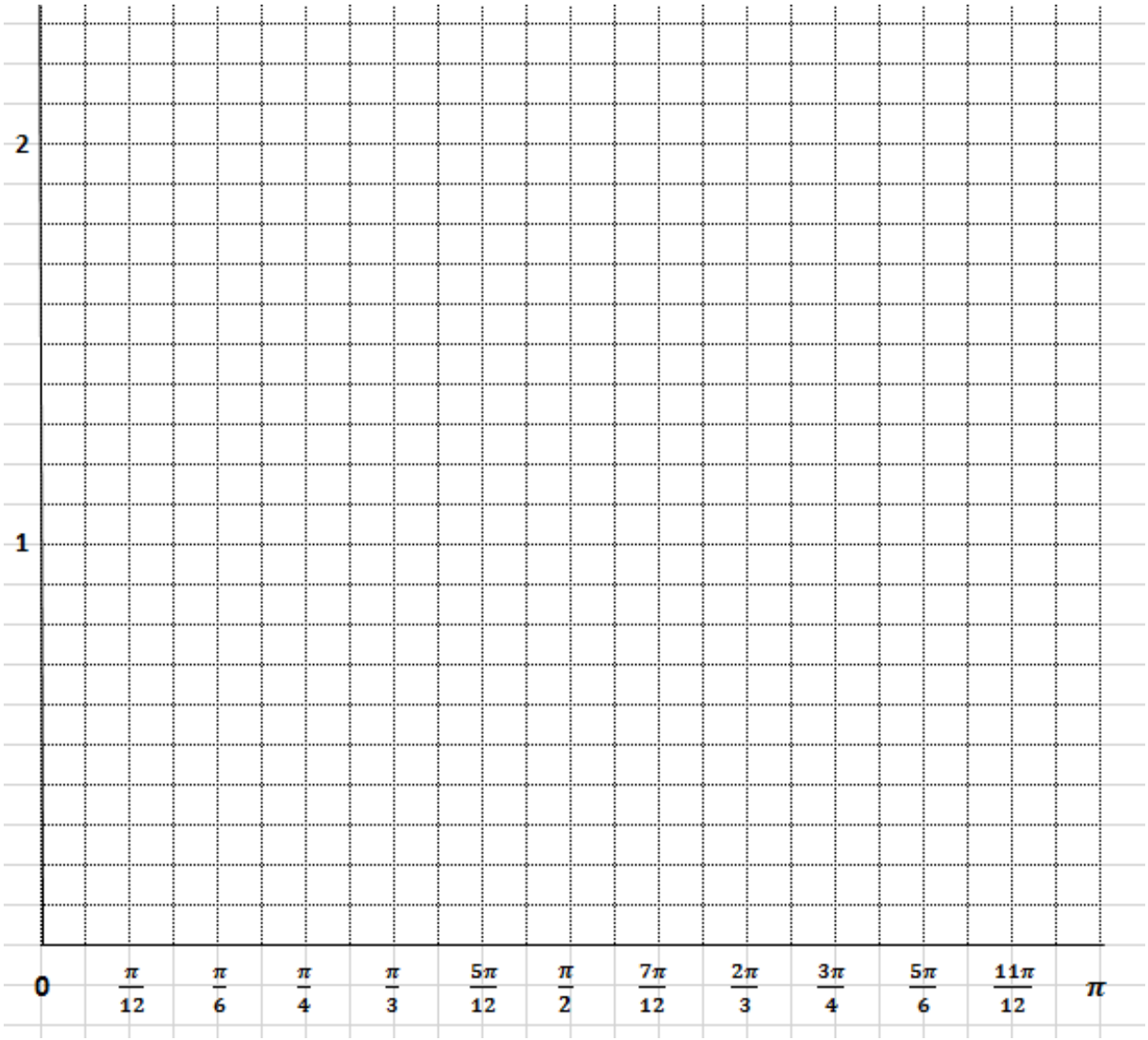
10. Given $f(t) = t + \cos t$ for $0 \leq t \leq \pi$. where the first and the second derivatives of f are $f'(t) = 1 - \sin t$, and $f''(t) = -\cos t$, respectively.

10.1 Find the critical points of f. [2]

10.2 Determine the curvature. [2]

10.3 Determine the point of inflection. [1]

10.4 Sketch the graph of f [2]



[**TOTAL 40**]