

UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE



UNIVERSITEIT  
VAN  
JOHANNESBURG

**DEPARTMENT OF PURE AND APPLIED MATHEMATICS**

**MODULE**      **MAT1C3E (CALCULUS SECTION)**  
**BIO & ENVIRO MATHS AND STATS**

**CAMPUS**      **APK**

**EXAM**      **JUNE 2016**

**DATE:**      26 MAY 2016

**SESSION:** 08:30 – 10:30

**ASSESSOR:**

MR. T. MOHUBEDU

**INTERNAL MODERATOR:**

MR. V. VAN APPEL

**DURATION:** 60 MINUTES

**MARKS:** 40

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**SURNAME AND INITIALS:** \_\_\_\_\_

**STUDENT NUMBER:** \_\_\_\_\_

**CONTACT NUMBER:** \_\_\_\_\_

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**Please read the following instructions carefully**

1. Answer all questions on the paper in pen.
2. This paper consists of 9 pages including the cover page.
3. Show all calculations.
4. Calculators are allowed.

1. Use the binomial Theorem to expand  $(2x + y)^3$ . [2]

2. Given  $f(x) = \cos x + \ln x$

2.1 Find the first derivative of  $f$ . [1]

2.2 Find the second derivative of  $f$ . [1]

3. Find  $f'$  given  $f(x) = \ln(\cos 3x)$  [2]

4. Use the product rule for derivatives to find the derivative of [2]

$$f(x) = (x^2 - 3)(-2x^2 + 1)$$

5. Use the product rule for derivatives to find the derivative of [3]

$$f(x) = \frac{e^x}{1 + \sin x}$$

6. Given  $x^2 + xy - y^2 + 1 = 0$

6.1 Use implicit differentiation to find  $y'$ . [3]

6.2 Find the slope of the tangent line(s) to  $f$  at  $x = 0$  [3]

7. An object is tossed upward at  $10 \text{ m/s}$  from a height of  $150 \text{ m}$ . The distance above the ground is given by  $M(t) = 150 + 10t - 4.9t^2$ .

7.1 Find the time when the object reaches a critical point. [2]

7.2 Find the maximum height of the object. [1]

7.3 Find the time when the object hits the ground. [2]

7.4 Find the objects' speed when it hits the ground. [1]

8. The mass  $M$  of an object is the product of density  $\rho$  and the volume  $V$ . If the mass and the volume of an insect are given by  $M(t) = 3 + t^2$  and  $V(t) = 1 + t^2$  for  $0 \leq t \leq 3$ .

8.1 Find the equations for the density  $\rho$  of the insect as a function of time. [1]

8.2 Find the derivative of the density of the insect [2]

9. Sketch the graph of any function with a negative second derivative. [1]



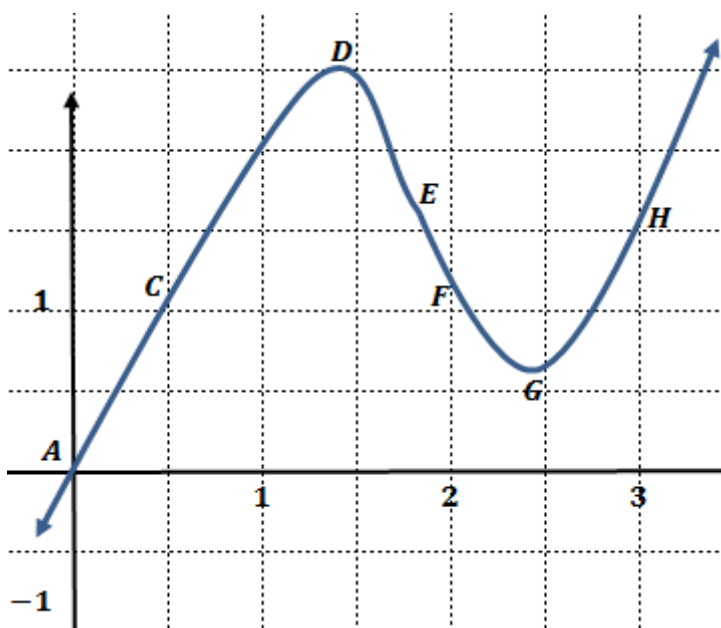
10. Which point on the given graph is:

10.1 a critical point. [1]

10.2 a point of inflection. [1]

10.3 a point with a negative derivative [1]

10.4 a point with a positive second derivative. [1]



11. Given  $f(t) = t + 2 \cos t$  for  $0 \leq t \leq \pi$ , where the first and the second derivatives of  $f$  are  $f'(t) = 1 - 2 \sin t$  and  $f''(t) = -2 \cos t$  respectively.

11.1 Find the critical points of  $f$ . [3]

11.2 Determine the curvature. [3]

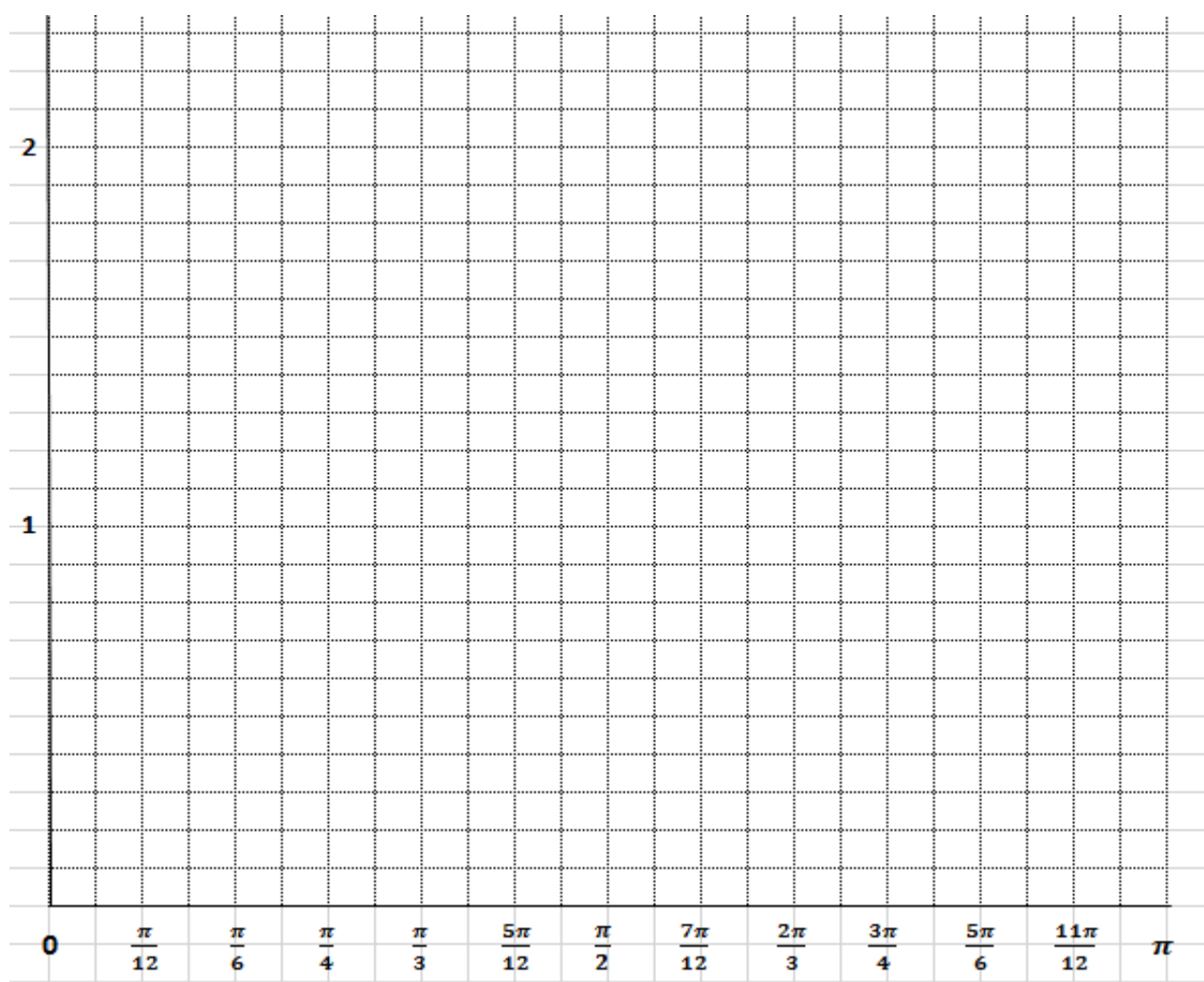


11.3 Determine the point of inflection.

[1]

11.4 Sketch the graph of  $f$

[2]



[ TOTAL 40 ]