UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

UNIVERSITEIT VAN JOHANNESBURG

DEPARTMENT OF PURE AND APPLIED MATHEMATICS

MODULE **MAT1C2E** (CALCULUS SECTION)
BIO & ENVIRO MATHS AND STATS

CAMPUS **APK**

EXAM **NOVEMBER 2016**

DATE: 30 NOVEMBER 2016 **SESSION:** 12:30 – 14:30

ASSESSOR: MR. T. MOHUBEDU

INTERNAL MODERATOR: MR. V. VAN APPEL

DURATION: 60 MINUTES **MARKS:** 40

SURNAME AND INITIALS: _____

STUDENT NUMBER: _____

CONTACT NUMBER: _____

Please read the following instructions carefully

1. Answer all questions on the paper in pen.
2. This paper consists of 9 pages including the cover page.
3. **Show all calculations.**
4. **Calculators are allowed.**

1. Given $f(x) = x^2 + 1$ and $g(x) = \sqrt{1 - x}$.

1.1 Does the point $(-1, 0)$ lie on the graph of f? [1]

1.2 Find the product $f.g$ [2]

1.3 Find the inverse of g. [2]

2. Use the laws of logarithm to simplify: $\log_2 3 - \log_2 4 - \log_2 6$ [2]

3. Find the equation of the straight line that is passing through the points $(-2,\ 1)$ and $(0, -3)$. [3]

4. Given $f(x) = 4 - x^2$

4.1 Find $f'(x)$ [1]

4.3 Give the interval of increase and decrease. [2]

5. Set up a table to estimate the limit: $\lim_{t \to 0} \frac{\sin(t)}{2t}$ [2]

6. The temperature of a room (T) is a function of how far the window is open (W, $in\ \ cm^2$) according to T (W)$= 34- 2.5$W . How long you sleep (S, measured in hours) is a function of the temperature according to $S(T) = 16.5 - 0.4T$

6.1 What is the maximum temperature of the room? [1]

6.2 Find the formula of how long you sleep as a function of how far the window is open. [2]

7. Consider the population $V(t)$ of viruses $(in\ millions)$ given by $V(t) = 15.0e^{0.65t}$ where time t is measured in hours.

7.1 Calculate the time at which the number of viruses will double in size. [3]

7.2 Find the equation of the line $\ln\big(V(t)\big)$ after transforming the variables to create a semilog plot. [2]

8. A population follows the discrete – time dynamical system $b_{t+1} = rb_t$

with $r = 0.75$ and $b_0 = 8.5$.

8.1 Show that the solution of the system is $b(t) = 8.5e^{-0.288t}$. [1]

8.2 Sketch the graph of the solution $b(t)$ for $0 \leq t \leq 5$. [3]

9. A population has a half – life of 4 years and an initial size of 5×10^3.

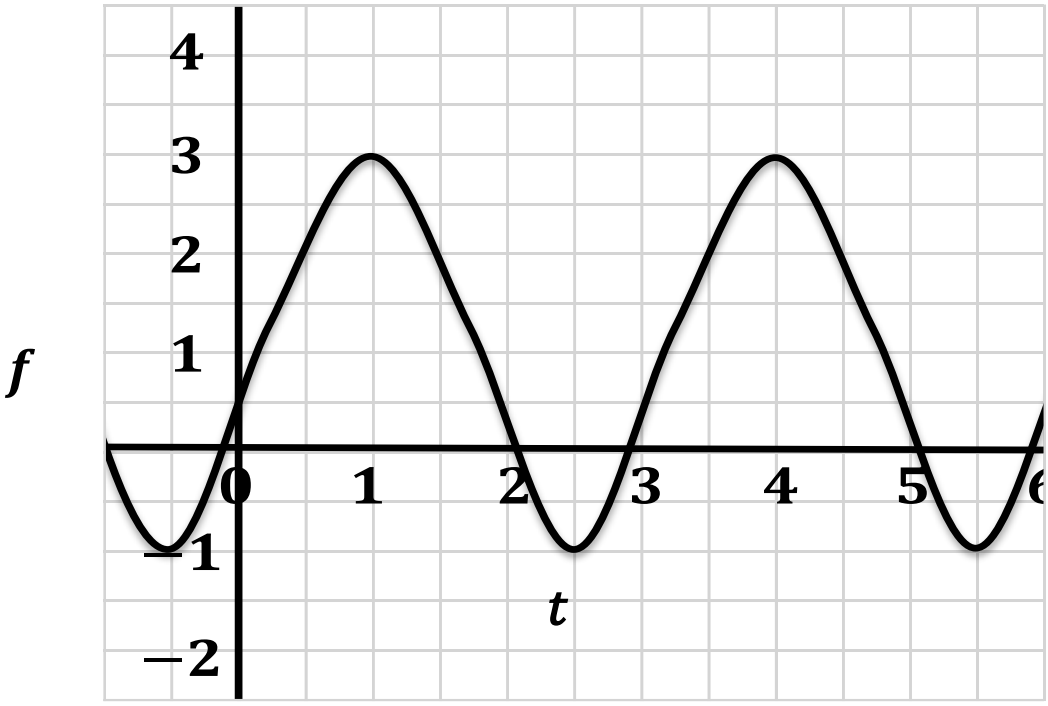
9.1 What is the population in 12 years? [1]

9.2 Find the equation for population size $P(t)$ as a function of time. [3]

10. Suppose the size (in cm) of an organism at time t (in $hours$) is given by $S(t) = 1.5\, e^t$.

Find the average rate of change in size during the second hour. [2]

11. Find the average, amplitude, period, phase and the equation of the given sinusoidal graph of f. [3]



12. Given $h(t) = 2 + 3\cos\left(\frac{\pi t}{2} - 0.786\right)$

12.1 Write h in standard form [1]

12.2 Sketch the graph of h for $0 \leq t \leq 7$. [3]

--------- [TOTAL 40] --------