## FACULTY OF SCIENCE

| DEPARTMENT OF PURE AND APPLIED MATHEMATICS |  |  |
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| MODULE | MAT1B01 <br> APPLICATIONS OF CALCULUS |  |
| CAMPUS APK <br> EXAM NOVEMBER EXAM 2016 |  |  |

DATE 28/11/2016
ASSESSOR(S)

INTERNAL MODERATOR DURATION 2 HOURS

SESSION 16:30-18:30

DR A CRAIG
MS S RICHARDSON
DR J MBA
MARKS 70

SURNAME AND INITIALS $\qquad$

STUDENT NUMBER $\qquad$

CONTACT NUMBER $\qquad$

NUMBER OF PAGES: $1+12$ PAGES
INSTRUCTIONS: 1. ANSWER ALL THE QUESTIONS ON THE PAPER IN PEN.
2. NO CALCULATORS ARE ALLOWED.
3. SHOW ALL CALCULATIONS AND MOTIVATE ALL ANSWERS.
4. IF YOU REQUIRE EXTRA SPACE, CONTINUE ON THE

ADJACENT BLANK PAGE AND INDICATE THIS CLEARLY.

## Question 1 [10 marks]

For questions 1.1 - 1.10, choose one correct answer, and make a cross (X) in the correct block.

| Question | a | b | c | d | e |
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| 1.1 |  |  |  |  |  |
| 1.2 |  |  |  |  |  |
| 1.3 |  |  |  |  |  |
| 1.4 |  |  |  |  |  |
| 1.5 |  |  |  |  |  |
| 1.6 |  |  |  |  |  |
| 1.7 |  |  |  |  |  |
| 1.8 |  |  |  |  |  |
| 1.9 |  |  |  |  |  |
| 1.10 |  |  |  |  |  |

1.1 Identify the definite integral that represents the area of the surface formed by revolving $y=\frac{1}{x}$ on the interval $[1,3]$ about the $x$-axis.
(a) $2 \pi \int_{1}^{3} \frac{1}{y} \sqrt{1+\frac{1}{y^{4}}} d y$
(b) $\int_{1}^{3} \sqrt{1+\frac{1}{x^{2}}} d x$
(c) $\int_{1}^{\frac{1}{3}} \sqrt{1+\frac{1}{y^{2}}} d y$
(d) $2 \pi \int_{1}^{3} \frac{1}{x} \sqrt{1+\frac{1}{x^{4}}} d x$
(e) None of the above
1.2 Which method should be used to solve the differential equation $\frac{d y}{d x}=\frac{y}{2 x}$ ?
(a) Linear equations
(b) Direct integration
(c) Separation of variables
(d) Bernoulli equations
(e) None of the above
1.3 The polar coordinate $\left(2 \sqrt{2}, \frac{3 \pi}{4}\right)$ converted to Cartesian coordinates is:
(a) $(-2,-2)$
(b) $\left(-2, \frac{1}{2}\right)$
(c) $(2,-2)$
(d) $(-2,2)$
(e) None of the above
1.4 Consider the integral $\int_{2}^{3} \frac{1}{x(x-2)} d x$. Which limit should be used to check whether the integral is divergent or convergent?
(a) $\lim _{t \rightarrow 2^{+}} \int_{2}^{t} \frac{1}{x(x-2)} d x$
(b) $\lim _{t \rightarrow 2^{+}} \int_{t}^{3} \frac{1}{x(x-2)} d x$
(c) $\lim _{t \rightarrow 0^{+}} \int_{t}^{3} \frac{1}{x(x-2)} d x$
(d) $\lim _{t \rightarrow 2^{-}} \int_{t}^{3} \frac{1}{x(x-2)} d x$
(e) None of the above
1.5 The critical numbers of $g(x)=\frac{x}{2}+\cos x$ on $[0, \pi]$ are:
(a) $\frac{\pi}{3}, \frac{2 \pi}{3}$
(b) $\frac{\pi}{6}, \frac{5 \pi}{6}$
(c) $\frac{\pi}{4}, \frac{3 \pi}{4}$
(d) $\frac{\pi}{3}, \frac{5 \pi}{6}$
(e) None of the above
1.6 The average value of $y=\cos x$ on the interval $[-3,5]$ is:
(a) $\frac{\sin 5-\sin 3}{2}$
(b) $\frac{\sin 3-\sin 5}{2}$
(c) $\frac{\sin 5+\sin 3}{8}$
(d) $\frac{\sin 3-\sin 5}{8}$
(e) None of the above
1.7 Find the equation of the directrix of the parabola $5 y=x^{2}$.
(a) $y=-\frac{5}{4}$
(b) $x=\frac{5}{2}$
(c) $y=-5$
(d) $x=0$
(e) None of the above
1.8 Let $f(x)$ be a function that is continuous on $[0,2]$ and differentiable on $(0,2)$. Then Rolle's Theorem guarantees that...
(a) there exists $c \in(0,2)$ such that $f^{\prime}(c)=f^{\prime}(0)=f^{\prime}(2)$.
(b) if $f(0)=f(2)$ then there exists $c \in[0,2]$ such that $f^{\prime}(c)=0$.
(c) there exists $c \in(0,2)$ such that $f(0)<f^{\prime}(c)<f(2)$.
(d) if $f(0)=f(2)$ then there exists $c \in(0,2)$ such that $f^{\prime}(c)=0$.
(e) None of the above
1.9 Find the slant asymptote of the curve $y=\frac{2 x^{3}-4 x^{2}-9}{x^{2}+3}$.
(a) $y=2 x+4$
(b) $y=-6 x+3$
(c) $y=2 x-4$
(d) $y=6 x-3$
(e) None of the above
$1.10 \sum_{k=0}^{n}\binom{n}{k} 9^{k}=\ldots$
(a) $10^{n}$
(b) $9^{n}$
(c) $9^{k}$
(d) $10^{k}$
(e) None of the above

## Question 2 [3 marks]

Sketch a function that satisfies the following properties:

- $f^{\prime}(x)>0$ on $(-\infty,-1) \cup(2, \infty), f^{\prime}(x)<0$ on $(-1,2)$
- $f^{\prime \prime}(x)>0$ on $(-\infty,-2) \cup(4, \infty), f^{\prime \prime}(x)<0$ on $(-2,2) \cup(2,4)$
- $\lim _{x \rightarrow 2^{-}} f(x)=-\infty$ and $\lim _{x \rightarrow 2^{+}} f(x)=-\infty$


## Question 3 [4 marks]

Prove the Mean Value Theorem. That is, prove the statement: "If a function $f(x)$ is continuous on $[a, b]$ and differentiable on $(a, b)$ then there exists $c \in(a, b)$ such that $f^{\prime}(c)=\frac{f(b)-f(a)}{b-a}$ ".

## Question 4 [2 marks]

We want to find the coordinates of the point on the curve $y=\sin x$ that is closest to the point $(4,2)$. Set up an equation for the distance from the curve to the point and differentiate your equation. You do not have to solve for the coordinates of the point.

Question 5 [4 marks]
(a) Use the Binomial Theorem to expand $(1-\sqrt{x})^{4}$.
(b) Hence find the coefficient of $x$ in $(\sqrt{x}+1)(1-\sqrt{x})^{4}$.

Question 6 [7 marks]
Evaluate the following integrals:
(a) $\int(\ln x)^{2} d x$
(b) $\int \frac{x^{2}}{\sqrt{4+x^{2}}} d x \quad\left(\right.$ Hint: $\left.\int \sec ^{3} u d u=\frac{1}{2}(\sec u \tan u+\ln |\sec u+\tan u|)+C\right)$

Question 7 [7marks]
(a) Use partial fractions to prove that: $\int \frac{d x}{a^{2}-x^{2}}=\frac{1}{2 a} \ln \left|\frac{a+x}{a-x}\right|+C$
(b) For the improper integral $\int_{3}^{\infty} \frac{1}{x^{3}} d x$, determine if it converges or diverges. If it converges, calculate the value.

Question 8 [6 marks]
Let $R$ be the region bounded by the graphs $y=\sqrt{x-1}$ and $y=(x-1)^{2}$.
(a) Set up a integral for the area of the region $R$ in terms of $\mathbf{y}$.
(b) Set up an integral for the volume of the solid generated by rotating $R$ about the line $x=-1$ using the method of cylindrical shells.

Question 9 [4 marks]
Find the arc length of $f(x)=\frac{1}{12} x^{3}+x^{-1}$ on $[1,2]$.

Question 10 [4 marks]
Find the solution for the differential equation $e^{x^{2}} y y^{\prime}+x=0$ if $y(0)=2$.

Question 11 [5 marks]
(a) Sketch the curve defined by the parametric equations and indicate the direction with an arrow:
(b) Find an equation of the tangent line to the curve $x=e^{\sqrt{t}}$ and $y=t-\ln \left(t^{2}\right)$ at $t=1$.

Question 12 [7 marks]
Consider the polar equation $r=2+\cos 2 \theta$.
(a) Sketch the graph of the given cardioid.
(b) Describe the symmetry of the given cardioid about the polar axis.
(c) Set up the integral, but do not evaluate, of the area that lies outside the given cardioid and inside $r=2$.

Question 13 [7marks]
(a) What conic is given by $2 x(6-x)=y(8+y)$ ? Give a full description.
(b) Sketch the conic $\frac{(x-3)^{2}}{25}-\frac{(y+1)^{2}}{49}=1$ clearly labelling the vertices and asymptotes. (3)

