UNIVERSITY OF JOHANNESBURG
FACULTY OF SCIENCE

Depertment of Pure and Applied Mathematics
APK Campus
MAT1A3E
Calculus of One-Variable Functions B
May/June 2016 Exam

EXAMINER:	Mr. JM Homann
SECOND EXAMINER:	Mr. ST Mohubedu
Moderator:	Dr. FP Schulz
TIME: 2 Hours	**Marks:** 50

Surname and Initials:

Student Number:

Contact Number:

Instructions:

1. Write your answers after the questions.
2. If you need more writing space then clearly indicate where your solution continues (start with the back of the previous page).
3. No calculators are permitted.
4. Write your answers in blue or black ink; anything in pencil will not be marked.
5. This exam consists of 14 pages, including this one and 6 blank pages (pages 9-14).

Question 1

Evaluate the following

a)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\sinh\left(x\right)\right] \qquad \text{(3 marks)}$$

b)

$$\lim_{p\to\infty}\frac{1}{p^4}\sum_{n=1}^{p}\left(n+n^3\right) \qquad \text{(3 marks)}$$

Question 2

Express the following Riemann Sums as definite integrals:

a)

$$\lim_{N\to\infty} \sum_{n=1}^{N} \left[\frac{y_n^2 e^{-y_n^2} \tanh^3(y_n)}{\ln(y_n^6) \tan^{-1}(y_n)} \right] \Delta y \qquad \text{(2 marks)}$$

on $[a, b]$, where a and b are some real numbers.

b)

$$\lim_{k\to\infty} \sum_{j=1}^{k} \left[1 + \cos\left(z_j^6\right) - \frac{z_j^2 + 5}{z_j^3 + 1} \right]^{z_j} \Delta z \qquad \text{(2 marks)}$$

on $[0, 5]$.

Question 3
With appropriate reasoning, state whether or not the following integrals exist.

a)
$$\int_{-1}^{1} \frac{x+6}{x^2-1}\ \mathrm{d}x. \qquad \text{(2 marks)}$$

b)
$$\int_{-1}^{1} \ln\left(|z| + 5z^2 + 5\right)\ \mathrm{d}z. \qquad \text{(2 marks)}$$

Question 4
Determine the following derivatives.

a)

$$\frac{\mathrm{d}}{\mathrm{d}y}\left[\int_0^{\ln\left(y^2\right)} \frac{|z|-5}{\cos^2\left(z^3\right)+3}\,\mathrm{d}z\right] \qquad \text{(2 marks)}$$

b)

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\int_{\frac{1}{t^2}}^{te^{t\ln(2t)}} \left(\sec\left(r\right)\cot\left(r\right)-7r^r\right)\,\mathrm{d}r\right] \qquad \text{(4 marks)}$$

Question 5

A toxin is released into a greenhouse and has a concentration (measured by the unit M), of $T(t)$ at any point in time, t, where t is measured in seconds (s). If the the toxin's concentration changes at a rate of

$$t - \frac{6}{t} - 5 \quad \mathrm{Ms}^{-1},$$

then, from 1 s after the toxin was released, determine the following after 10 s:

a) The net change in the toxin's concentration;

(4 marks)

b) The total change in the toxin's concentration.

(3 marks)

Question 6

Evaluate the following integrals.

a)

$$\int_0^{\frac{T}{2}} \left[-5\cos^2\left(\frac{2\pi t}{T-\alpha}\right) \sin\left(\frac{2\pi t}{T-\alpha}\right)\right] \, \mathrm{d}t, \qquad \text{(6 marks)}$$

where $\alpha \in \mathbb{R}$ is constant.

b)

$$\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \left[\cos(x)\, e^{\sin(x)} + x^3\right] \, \mathrm{d}x \qquad \text{(8 marks)}$$

reduce you answer for (b) to a single function.

Question 7

a) State the Substitution Rule for Definite Integrals.

(1 mark)

b) State the Fundamental Theorem of Calculus Part II.

(1 mark)

c) Prove the Fundamental Theorem of Calculus Part II.

(7 marks)