UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS	
MODULE	MAT1A01 CALCULUS OF ONE-VARIABLE FUNCTIONS
CAMPUS	APK
EXAM	JUNE EXAM 2016

DATE 31/05/2016	SESSION 12:30 - 14:30
ASSESSOR(S)	DR A CRAIG MS S RICHARDSON
INTERNAL MODERATOR	DR J MBA
DURATION 2 HOURS	MARKS 70

SURNAME AND INITIALS ______________________________

STUDENT NUMBER ______________________________

CONTACT NUMBER ______________________________

NUMBER OF PAGES: 1 + 12 PAGES

INSTRUCTIONS:
1. ANSWER ALL THE QUESTIONS ON THE PAPER IN PEN.
2. NO CALCULATORS ARE ALLOWED.
3. SHOW ALL CALCULATIONS AND MOTIVATE ALL ANSWERS.
4. IF YOU REQUIRE EXTRA SPACE, CONTINUE ON THE <u>ADJACENT</u> BLANK PAGE AND INDICATE THIS CLEARLY.

Question 1 [8 marks]

For questions 1.1 - 1.8, choose **one** correct answer, and make a cross (X) in the correct block.

Question	**a**	**b**	**c**	**d**	**e**
1.1					
1.2					
1.3					
1.4					
1.5					
1.6					
1.7					
1.8					

1.1 Which one of the following is a negation of [1]

"Tim is inside and Leo is at the pool."

a) Tim is inside or Leo is not at the pool.

b) Tim is inside or Leo is at the pool.

c) Tim is not inside or Leo is at the pool.

d) Tim is not inside and Leo is not at the pool.

e) Tim is not inside or Leo is not at the pool.

1.2 What is the correct translation of the sentence into First Order language: [1]

"The square of x is greater than x whenever x is any positive real number."

a) $\exists x \in \mathbb{R}((x > 0) \wedge (x^2 > x))$

b) $\forall x \in \mathbb{R}((x > 0) \rightarrow (x^2 > x))$

c) $\forall x \in \mathbb{R}((x > 0) \wedge (x^2 > x))$

d) $\exists x \in \mathbb{R}((x \geq 0) \rightarrow (x^2 \geq x))$

e) None of the above

1.3 If a function $g(x)$ is shifted 1 unit to the right, then shifted 4 units down, and then reflected about the x-axis, the resulting equation is: [1]

a) $g(x+1)-4$

b) $-g(x-1)-4$

c) $-g(x-1)+4$

d) $g(x+1)+4$

e) None of the above

1.4 If $3 \leqslant f(x) \leqslant 7$ for $-1 \leqslant x \leqslant 1$, then which of the following is guaranteed to be true: [1]

a) $-3 \leqslant \displaystyle\int_{-1}^{1} f(x)\,dx \leqslant 7$

b) $-6 \leqslant \displaystyle\int_{-1}^{1} f(x)\,dx \leqslant 7$

c) $3 \leqslant \displaystyle\int_{-1}^{1} f(x)\,dx \leqslant -7$

d) $6 \leqslant \displaystyle\int_{-1}^{1} f(x)\,dx \leqslant 14$

e) None of the above

1.5 Which pair correctly fills the blanks in the following theorem: [1]

If f is continuous on ____ then $\displaystyle\int_{a}^{b} f(x)\,dx = F(b)-F(a)$ where F is a function such that ____.

a) (a,b), $F' = f$

b) $[a,b]$, $f' = F$

c) $[a,b]$, $F' = f$

d) (a,b), $f' = F$

e) None of the above

1.6 If $y = (\ln x)^3$, then $\dfrac{dy}{dx} =$ [1]

a) $\frac{3}{x}(\ln x)^2$

b) $3(\ln x)^2$

c) $3x(\ln x)^2 + (\ln x)^3$

d) $3(\ln x + 1)$

e) None of the above

1.7 If $F(x) = x \sin x$, then $F'(\frac{3\pi}{2}) =$ [1]

a) 0

b) 1

c) -1

d) $-\frac{3\pi}{2}$

e) None of the above

1.8 All the functions below, except one, has the property that $f(x)$ is equal to its fourth derivative. Which one of the following does not have this property? [1]

a) $f(x) = \sin x$

b) $f(x) = e^{2x}$

c) $f(x) = \cos x$

d) $f(x) = e^{-x}$

e) None of the above

Question 2 [7 marks]

a) Fill in the blank spaces in the following table. [3]

p	q	$\neg p$	$\neg q$	$\neg q \to p$	$p \wedge \neg p$
T	T				
T	F				
F	T				
F	F				

b) Answer the following **true/false** questions. For each item (i - iv) make a cross (X) in the corresponding block.

Question	**TRUE**	**FALSE**
i		
ii		
iii		
iv		

i) $\neg p \to q$ is the negation of $p \to q$. [1]

ii) $p \wedge \neg p$ is a contradiction. [1]

iii) $\neg q \to \neg p$ is the converse of $p \to q$. [1]

iv) $\neg q \to p$ is a tautology. [1]

Question 3 [6 marks]

a) Use proof by contrapositive to show that if $\sin\left(\frac{k\pi}{2}\right) \neq -1$, then $k \neq 7$. [2]

b) Solve the inequality and express your answer in interval notation:

$$\frac{2x}{2x-1} \leq 1$$

[2]

c) Prove the identity $\tan x + \dfrac{\cos x}{\sin x - 1} = -\sec x$ [2]

Question 4 [8 marks]

a) Determine the domain of the following function, and express your answer in interval notation:

$$f(x) = \sqrt{x-1} - \sqrt{2-x}$$

[2]

b) Determine the range of the function $g(x) = 3 + \sin(x)$, and express your answer in interval notation: [1]

c) Let:

$$f(x) = \begin{cases} \cos x & \text{if } x < 0 \\ \sqrt{x} & \text{if } x \geq 0 \end{cases}$$

d) Sketch the graph of f over the interval $[\frac{-\pi}{2}, 4]$. [2]

e) Calculate $\lim_{x\to 0} f(x)$. [2]

f) Is f continuous at $x = 0$? Explain your answer. [1]

Question 5 [5 marks]

Determine the following limits if they exist. Do not use L'Hospital's Rule.

a) $\lim_{x\to 3} \dfrac{\sqrt{x+6}-x}{x-3}$ [3]

b) $\lim_{x\to \infty} \dfrac{2x^2+x-9}{x^2-x+3}$ [2]

Question 6 [5 marks]

a) Suppose $g(x) = \displaystyle\int_2^{\sin x} e^t\,dt$. Use the Fundamental Theorem of Calculus (Part 1) to find $g'(x)$. [2]

b) Use the information provided to find $f(t)$: $f''(t) = \cos t,\ f'(0) = 3,\ f(0) = 5.$ [3]

Question 7 [7 marks]

Evaluate the following integrals:

a) $\displaystyle\int_{-1}^{2} (3x^2 + 3x + 3)\, dx$ [3]

b) $\displaystyle\int \left(\frac{5^x}{2} + \frac{3}{1+x^2}\right) dx$ [2]

c) $\int xe^{x^2}\,dx$ [2]

Question 8 [7 marks]

a) Find $\frac{d}{dx}\left(\log_3\left(\cot x + 2^{3x^2}\right)\right)$ [2]

b) Find the slope of the tangent line to the curve at the given point: [3]

$$(x^2+y^2)^2 = 4x^2y \qquad (1,1)$$

c) Prove that $\dfrac{d}{dx}\left(\csc^{-1} x\right) = \dfrac{-1}{x\sqrt{x^2-1}}$ [2]

Question 9 [3 marks]

Use logarithmic differentiation to find y' if $y = \dfrac{(e^{2x}+6)^7.\sqrt{x+4}}{(e^{-x}+e^{x})^5}$ [3]

Question 10 [4 marks]

Use mathematical induction to prove that for all $n \geqslant 1$:

$$\sum_{j=1}^{n} \frac{1}{(2j-1)(2j+1)} = \frac{n}{2n+1}$$

[4]

Question 11 [4 marks]

If f and g are both differentiable and $g(x) \neq 0$, prove the Quotient Rule of differentiation, that is

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

[4]

Question 12 [6 marks]

a) Evaluate the following limit. Use L'Hospital's Rule if necessary: $\lim_{x \to 0} (\cos x)^{\frac{5}{x}}$ [3]

b) If $y = \ln(\sec x + \tan x)$, use the definition of the hyperbolic function to show that $\cosh y = \sec x$. [3]