UNIVERSITY OF JOHANNESBURG

# FACULTY OF SCIENCE

## DEPARTMENT OF PURE AND APPLIED MATHEMATICS

**MODULE:** **MAFT0A3/MA3AFET**

**COURSE:** **MATHEMATICS 3A FOR TEACHERS**

**CAMPUS:** **APK**

**EXAM:** **JUNE 2016**

**DATE:** **TUESDAY 7 JUNE 2016**

**TIME:** **08:30 – 11:30**

**ASSESSOR:** **MS. R. DURANDT**

**EXTERNAL MODERATOR:** **DR. B. POSTHUMA**

**DURATION:** **3 HOURS** **MARKS: 120**

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**SURNAME AND INITIALS**

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**STUDENT NUMBER**

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**CONTACT NUMBER**

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**NUMBER OF PAGES:** **16 PAGES (including front page)**

**INSTRUCTIONS:** **ANSWER ALL THE QUESTIONS, CALCULATORS ARE NOT ALLOWED**

**SECTION A: SHORT ANSWER AND THEORY QUESTIONS [10]**

**Question 1: (5)**

Complete the following short answer questions by providing the correct answer in the allocated space.

| Question | Answer |
|---|---|
| Evaluate the integral: $$\int x^3 dx$$ | |
| Evaluate the integral: $$\int \frac{dx}{x}$$ | |
| Evaluate the integral: $$\int e^{2x} dx$$ | |
| Evaluate the integral: $$\int sinx dx$$ | |
| Evaluate the integral: $$\int \frac{dx}{9 + x^2}$$ | |

**Question 2: (5)**

2.1 State the rule for integration by parts.

(1)

2.2 If the expression $\sqrt{\boldsymbol{a^2 - x^2}}$ occurs in an integral, what trigonometry substitution might you try?

(1)

2.3 Give the partial fraction decomposition for:

$$\frac{\boldsymbol{x^2 + 4}}{\boldsymbol{x^2(x - 4)}}$$

(1)

2.4 State **True** or **False**:

$$\boldsymbol{\cos^{-1} x = \frac{1}{cosx}}$$

(1)

2.5 State **True** or **False**:

$$\boldsymbol{\frac{d}{dx}(ln5) = \frac{1}{5}}$$

(1)

**SECTION B: MULTIPLE CHOICE QUESTIONS [20]**

**Question 3:**

The following questions are multiple choice questions. There is only one correct answer from the choices given. Select the correct option by marking the option with a cross (**X)**.

3.1. Find the value of the integral:

$$\int_1^e \frac{\boldsymbol{sec^2\left(\frac{\pi}{4}lnx\right)}}{\boldsymbol{x}}\boldsymbol{dx}$$

a) $\frac{4}{\sqrt{2}}$
b) $\frac{4}{\sqrt{2}\pi}$
c) $0$
d) $\frac{4}{\pi}$
e) None of the above.

(5)

3.2. Evaluate the following indefinite integral:

$$\int \frac{\boldsymbol{3dx}}{\boldsymbol{\sqrt{9x^2-1}}}$$

a) $sec\theta - tan\theta + C$
b) $sec\theta + tan\theta + C$
c) $ln|2sec\theta + tan\theta| + C$
d) $ln|sec\theta + tan\theta| + C$
e) None of the above.

(5)

3.3. If $\boldsymbol{g(x) = \int_1^{\sqrt{x}} \frac{e^t}{t} dt}$, then $\boldsymbol{g'(1)}$ is:

a) $e$
b) $e^2$
c) $\frac{e}{2}$
d) $\frac{e}{3}$
e) None of the above.

(5)

3.4. If $\boldsymbol{f(x) = x^{\sqrt{x}}}$ then $\boldsymbol{f'(4)}$ is

a) $2 + ln4$
b) $8 + 4ln4$
c) $16 + 4ln2$
d) $4 + ln2$
e) None of the above

(5)

**SECTION C: APPLICATIONS** **[90]**

**Question 4:** **(10)**

4.1 Express the given integral as a limit of Riemann sums (do not evaluate the limit):

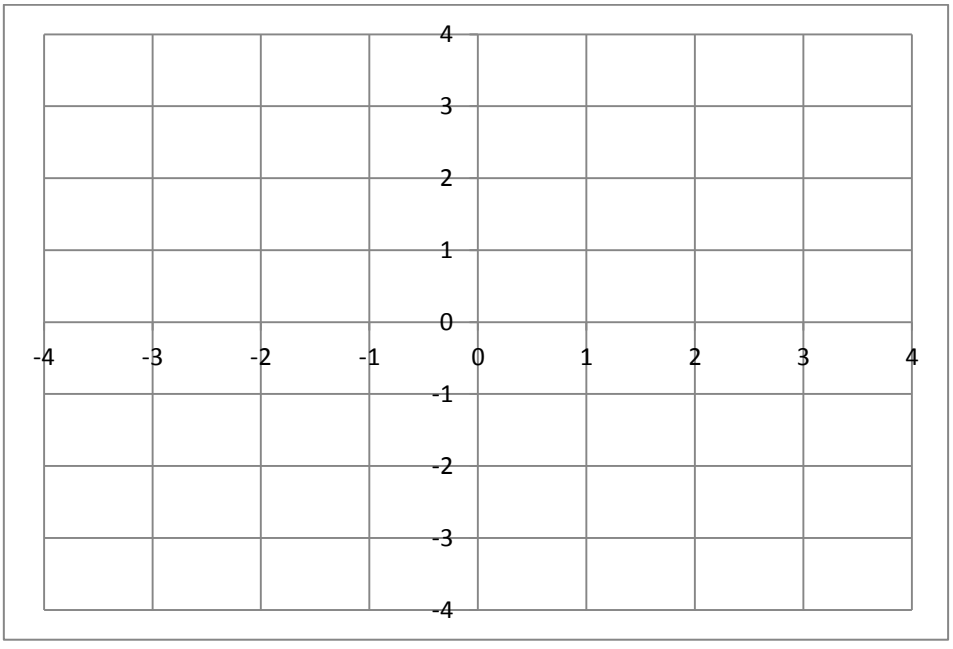
$$\int_0^3 (x^2 - 3x)dx$$

(2)

4.2 Evaluate the integral by interpreting it in terms of areas. Draw a diagram on the given grid to illustrate the interpretation:

$$\int_{-3}^{0} 1dx + \int_0^3 \sqrt{9 - x^2}dx$$

(3)



4.3 Given $\boldsymbol{g(x) = \int_0^x f(t)dt}$, where $f$ is the function whose graph is shown.

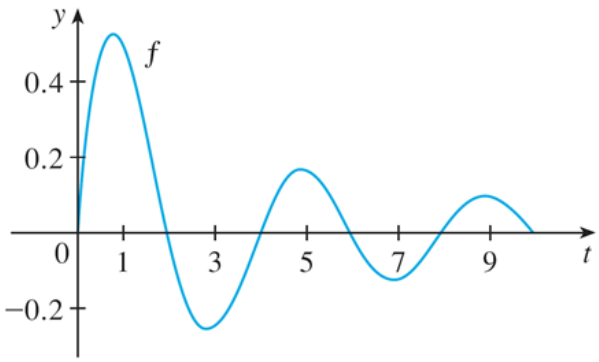


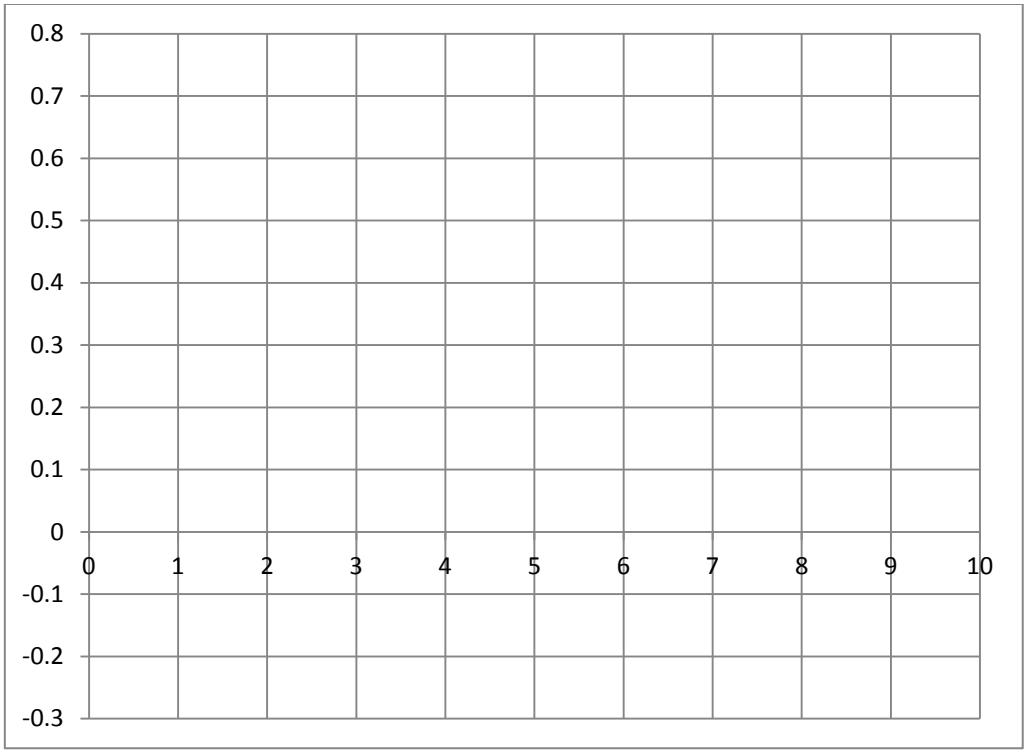

Figure from Stewart, J. (2013). Essential Calculus. CENGAGE Learning

4.3.1 Give four possible values of $x$ where the local maximum and minimum values of $g$ occur?

(2)

4.3.2 Sketch a rough drawing of the graph of $g$ on the grid below.

(3)



**Question 5:** **(10)**

In a research experiment, a population of fruit flies is increasing according to the law of exponential growth. After 2 days there are 100 flies, and after 4 days there are 300 flies.

5.1 Let $y$ be the number of flies at time $t$. Substitute the given information to set up a model for exponential growth.

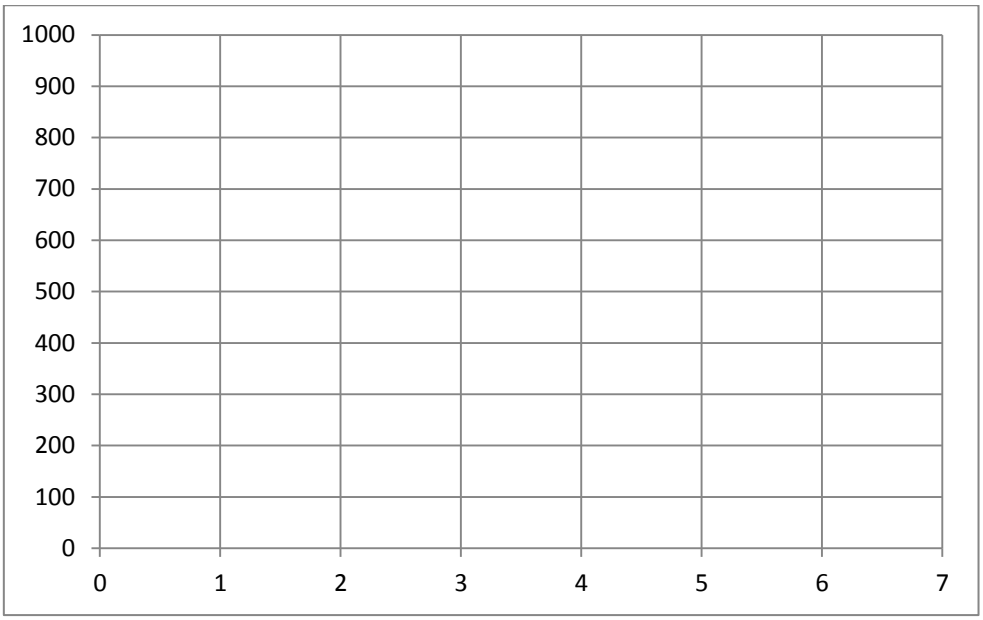
(5)

5.2 How many flies will there be after 6 days.

(2)

5.3 Use the information above to represent the model graphically (work accurately and neatly to show clear indications on the graph, use the set of axes provided).

(3)




**Question 6:** **(20)**

6.1 Differentiate the function (simplify your answer):

$$y = \frac{lnx}{x^2}$$

(3)

6.2 Evaluate the integral:

$$\int \frac{(\boldsymbol{lnx})^{\mathbf{2}}}{\boldsymbol{x}}\,\boldsymbol{dx}$$

(4)

6.3 Calculate the second derivative (simplify your answer):

$$\boldsymbol{f(x) = ln(x^2 + 10)}$$

(5)

6.4 Use logarithmic differentiation to find the derivative of the function:

$$\boldsymbol{y = \frac{(e^x + 1)^4 sin^2 x}{\sqrt{x}}}$$

(8)

**Question 7:** **(30)**

7.1 Differentiate the function (do not simplify your answer):

$$y = \tan^{-1} x\sqrt{1 - x^2}$$

(4)

7.2a) Write $x^2 + 2x + 5$ in the form $(x - p)^2 + q$.

(2)

b) Hence, evaluate the integral:

$$\int \frac{3dx}{x^2 + 2x + 5}$$

(4)

7.3 Find the limit:

$$\lim_{x \to 0^+} xlnx$$

(3)

7.4 Show by **differentiation** the following reduction formula is valid:

$$\int lnxdx = xlnx - x + C$$

(3)

7.5 Use the integration rule revealed in question 7.4 to evaluate the integral:

$$\int xe^{-x}dx$$

(5)

7.6a) Complete the formula:

$$cos2x = \_\_\_\_\_ = \_\_\_\_\_ = \_\_\_\_\_$$

(3)

b) Hence, evaluate the integral:

$$\int_0^{\pi/2} cos^2 x dx$$

(6)

**Question 8:** **(20)**

8.1 Determine the area $(A)$ included by the curves (draw a suitable graph and show clear calculations):

$$\boldsymbol{x = \frac{1}{2}y^2 \ \ and \ \ y = 2x - 2}$$

(6)

8.2 Calculate the volume obtained by rotating the closed region between the curve $\boldsymbol{y = x^4}$ and the line $\boldsymbol{y = 8x}$ about the line $\boldsymbol{x = 0}$. Draw a sketch, indicate the cross-sectional area and show all calculations.

(8)

8.3 Solve the differential equation below with $\boldsymbol{y(0) = 4}$, given:

$$\frac{dy}{dx} - xy = x$$

(6)