FACULTY OF SCIENCE

DEPARTMENT OF PURE AND APPLIED MATHEMATICS

MODULE **ASME2A1**
SEQUENCES, SERIES AND VECTOR CALCULUS

CAMPUS **APK**

EXAM **NOVEMBER 2016**

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DURATION	2.5 HOURS
MARKS	50

SURNAME AND INITIALS _____

STUDENT NUMBER _____

IDENTITY NUMBER _____

NUMBER OF PAGES: 1 + 11

INSTRUCTIONS:
1. ANSWER ALL QUESTIONS ON THE PAPER IN PEN
2. CALCULATORS ARE ALLOWED
3. INDICATE **CLEARLY** ANY ADDITIONAL WORKING OUT

Question 1

Determine whether the sequence is increasing, decreasing, or not monotonic. Is the sequence bounded? If it is convergent, find its limit. [5]

$$a_n = \ln\left(\frac{2n-1}{3n+4}\right)$$

Question 2

State and prove the Alternating Series Test. [6]

Question 3

Test the following series for convergence or divergence: [8]

(3.1) $\sum_{n=1}^{\infty}(-1)^n\left(\arctan n - \frac{\pi}{2}\right)$ (4)

(3.2) $\sum_{n=1}^{\infty} b^{\ln n}$, where $b \in \mathbb{R}$ satisfies $0 < b < e^{-1}$. **Hint:** Show that the series is a p-series. (4)

Question 4

Find a power series representation for the function and determine the radius of convergence: [4]

$$f(x) = \left(\frac{x}{1-x}\right)^3.$$

Question 5 [6]

(5.1) Use the Maclaurin series expansion of e^x to find a Maclaurin series expansion for (4)

$$f(x) = \cosh x.$$

(5.2) Why can we conclude from (5.2) that $f(x) \geq 1 + \frac{x^2}{2} + \frac{x^4}{24}$ for all $x \in \mathbb{R}$? (2)

Question 6

For questions (6.1) - (6.5), please circle only **ONE** correct answer: [5]

(6.1) Two particles travel along the space curves

$$\mathbf{r}_1(t) = \left\langle \cos t, \sin^2 t, t^2 - 4\pi^2 \right\rangle \text{ and } \mathbf{r}_2(t) = \left\langle 1 + t, t^2, t^3 \right\rangle.$$

What can be said about the motion of the particles? (1)

(a) The particles collide.

(b) The paths of the particles intersect, but they do not collide.

(c) The paths of the particles do not intersect.

(d) None of the above.

(6.2) If $\mathbf{r}'(t) = \langle t^2, t\cos\pi t, \sin\pi t\rangle$ and $\mathbf{r}(0) = \left\langle 0, \frac{1}{\pi^2}, -\frac{1}{\pi}\right\rangle$, then (1)

(a) $\mathbf{r}(t) = \left\langle \frac{t^3}{3}, \frac{t}{\pi}\sin\pi t + \frac{1}{\pi^2}\cos\pi t, -\frac{1}{\pi}\cos\pi t\right\rangle$.

(b) $\mathbf{r}(t) = \left\langle \frac{t^3}{3}, \frac{t}{\pi}\cos\pi t - \frac{1}{\pi^2}\sin\pi t, -\frac{1}{\pi}\cos\pi t\right\rangle$.

(c) $\mathbf{r}(t) = \left\langle \frac{t^3}{3}, -\frac{t}{\pi}\sin\pi t - \frac{1}{\pi^2}\cos\pi t, \frac{1}{\pi}\cos\pi t\right\rangle$.

(d) $\mathbf{r}(t) = \left\langle \frac{t^3}{3}, \frac{t}{\pi}\sin\pi t - \frac{1}{\pi^2}\cos\pi t, \frac{1}{\pi}\cos\pi t\right\rangle$.

(6.3) If $\mathbf{r}(t)$ is a differentiable vector function, then (1)

$$\frac{d}{dt}|\mathbf{r}(t)| = |\mathbf{r}'(t)|.$$

(a) True (b) False

(6.4) Suppose that $\mathbf{u}$ and $\mathbf{v}$ are differentiable vector functions, and that f is a differentiable real-valued function. Consider the following differentiation formulas:

(i) $\frac{d}{dt}[\mathbf{u}(t)\cdot\mathbf{v}(t)] = \mathbf{v}'(t)\cdot\mathbf{u}(t) + \mathbf{u}'(t)\cdot\mathbf{v}(t)$.

(ii) $\frac{d}{dt}[\mathbf{u}(t)\times\mathbf{v}(t)] = \mathbf{v}'(t)\times\mathbf{u}(t) + \mathbf{u}'(t)\times\mathbf{v}(t)$.

(iii) $\frac{d}{dt}[\mathbf{u}(f(t))] = f'(t)\mathbf{u}'(f(t))$.

Which of the above statements are always true? (1)

(a) i, ii, iii (b) i, ii (c) iii (d) i, iii (e) ii, iii

(6.5) The curvature of $y = e^x$ at $x = 0$ is (1)

(a) $\frac{1}{2\sqrt{2}}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{1}{2}$ (d) $\frac{1}{2\sqrt{3}}$ (e) 2

Question 7

Find the length of the curve

$$\mathbf{r}(t) = \left\langle \frac{t^3}{3}, \frac{t^2}{\sqrt{2}}, t \right\rangle$$

from $t = 0$ to $t = 3$. [4]

Question 8

Show that if there is a $c \in \mathbb{R}$ such that $|\mathbf{r}(t)| = c$ for all t, then $\mathbf{r}'(t)$ is orthogonal to $\mathbf{r}(t)$. [3]

Question 9

Prove that the curvature of a curve C with vector function $\mathbf{r}(t)$ is given by the following formula:[4]

$$\kappa(t) = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}.$$

Question 10

Find the tangential and normal components of the acceleration vector of a particle with position function $\mathbf{r}(t) = \langle t, 2t, t^2 \rangle$. [5]