UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS

MODULE: **ASMA2B4**

CAMPUS: **APK**

EXAM: **JUNE 2016**

DATE:	**7 JUNE 2016**	**SESSION: 12:30 - 15:30**
ASSESSOR(S):	**C MARAIS**	
INTERNAL MODERATOR:	**E JOUBERT**	
DURATION:	**2 HOURS**	**MARKS: 50**

SURNAME AND INITIALS:

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STUDENT NUMBER:

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CONTACT NR:

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NUMBER OF PAGES: 9 PAGES

INSTRUCTIONS: **ANSWER ALL THE QUESTIONS IN PEN**
YOU MAY NOT USE A CALCULATOR
GOOD LUCK!

Question 1

Answer the following True or False questions and give a short justification/counter-example:

a) S_5 has an element of order 6. [2]

TRUE	
FALSE	

b) The cyclic group $\mathbb{Z}_{18}/\langle 6\rangle$ is isomorphic to $\mathbb{Z}_6$. [2]

TRUE	
FALSE	

c) If G and H are cyclic groups, then $G\times H$ is cyclic. [2]

TRUE	
FALSE	

d) If φ is a homomorphism with $Ker\varphi=\{e\}$ then φ is 1-to-1. [2]

TRUE	
FALSE	

<u>Question 2</u>

a) Let G be an Abelian group and let $H=\{a\in G : a^2=e\}$ be a subset of G. Use the Two-Step-Subgroup Test to show that H is a subgroup of G. [5]

b) Show that H as defined in a) is not a subgroup of G if G is not Abelian by giving a counter-example. [2]

Question 3

Let $G=(\mathbb{Z},+)$ and let $G'=(\{n\in\mathbb{Z}:n \text{ is even}\},+)$. Show that G and G' are isomorphic by showing that the function $\varphi:G\to G'$ defined by $\varphi(n)=2n$ is an isomorphism. [3]

Question 4

Let $G=GL(2,\mathbb{Q})$, the set of all invertible 2x2 matrices with rational entries and with matrix multiplication as operation. Let $H=\left\{\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}:a\in\mathbb{Q}\right\}$ be a subgroup of G. Show that H is not a normal subgroup of G, i.e. find a $g\in G$ such that $ghg^{-1}\notin H$ for some $h\in H$. [2]

<u>Question 5</u>

Let $G = \left\{ \begin{bmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix} \in M_{33} : a,b,c \in \mathbb{R} \right\}$ be a subgroup of $GL(3,\mathbb{R})$ (with matrix multiplication as operation).

a) Show that the function $\varphi\left(\begin{bmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix} \right) = (a,c)$ is a group homomorphism from G to $\mathbb{R} \oplus \mathbb{R}$ (with component-wise addition as operation). [3]

b) Find the kernel of φ. [2]

Question 6

Let $\varphi : G \to G'$ be a one-to-one homomorphism. Show that if $y^3 = e$ for all $y \in G'$, then $x^3 = e$ for all $x \in G$. [3]

Question 7

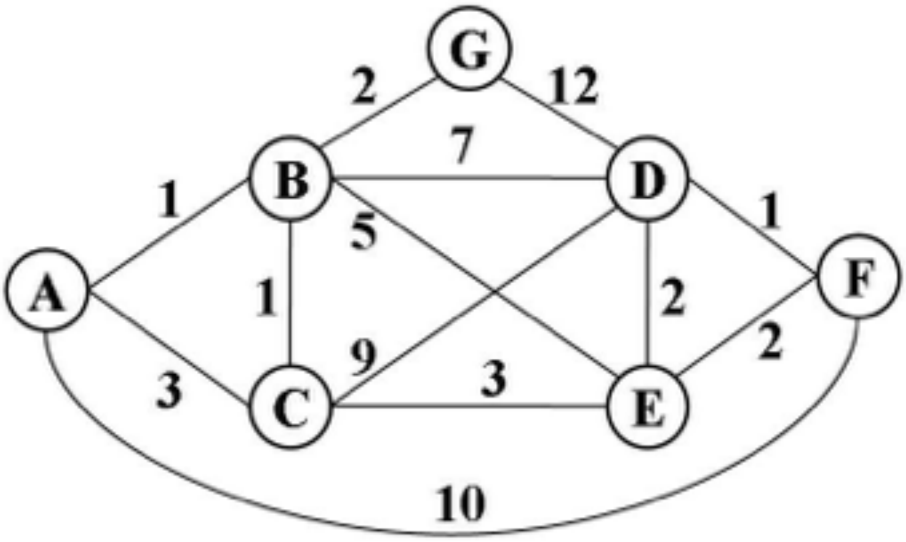
a) Let H be a subgroup of a group . Prove that H is normal in G if and only if $xHx^{-1} \subseteq H$ for $x \in G$. [4]

b) Let φ be a homomorphism from a group G_1 to a group G_2. Given that $Ker\varphi$ is a subgroup of G_1, prove that $Ker\varphi$ is normal. [3]

c) Let φ be a homomorphism from a group G_1 to a group G_2, and $\psi : G_1 / Ker\varphi \to \varphi(G_1)$ an isomorphism. Show that ψ is well-defined. [3]

<u>Question 8</u>

Consider the graph below and answer the following questions:



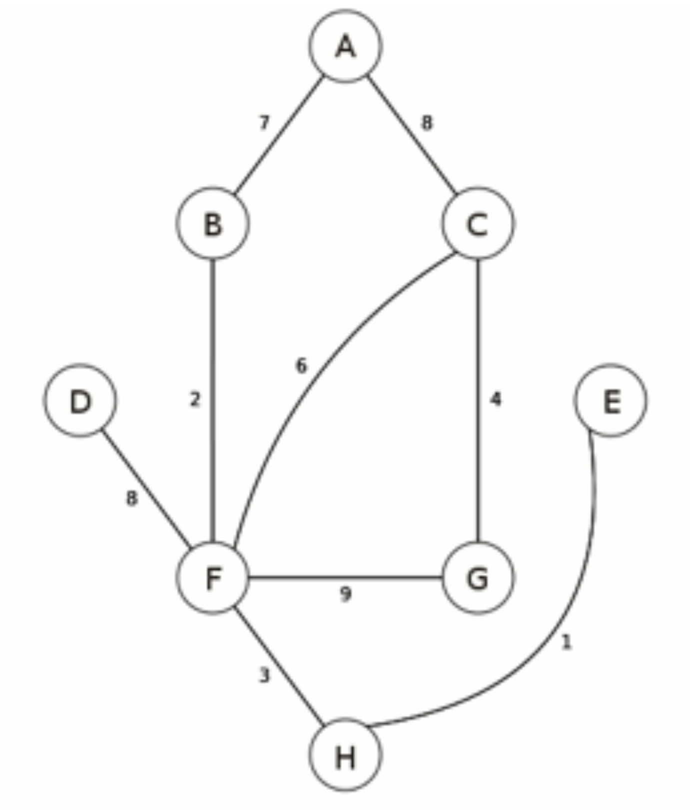
a) Use the Floyd-Warshall algorithm to determine which vertices are connected and to find the shortest paths for these vertices. You may use the table below for your final answer. [4]

	A	B	C	D	E	F	G
A							
B							
C							
D							
E							
F							
G							

b) What is the shortest distance from A to F? Give the path to follow to obtain this distance. [2]

Question 9

Consider the graph below and answer the questions that follow:



a) Suppose that Dijksra's algorithm is run to determine the shortest path from A to H in the graph. Complete the following table: [4]

Step	A	B	C	D	E	F	G	H	Last Trans fer	Perm
Initial	$(-,0)$	$(-,\infty)$	$(-,\infty)$	$(-,\infty)$	$(-,\infty)$	$(-,\infty)$	$(-,\infty)$	$(-,\infty)$	A	{A}
1										
2										
3										
4										
5										
6										

b) Now, write down the shortest path from A to H as read from the table in a). What is the length of this path? [2]