(7 pages)

UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF PURE AND APPLIED MATHEMATICS

ASMA2B2-2

LINEAR ALGEBRA (ENGINEERING)

EXAMINATION **31 May 2016**

EXAMINER Prof H Raubenheimer

INTERNAL MODERATOR Ms C Marais

TIME ALLOWED: 2 HOURS **MARKS: 60**

SURNAME AND INITIALS: .....

STUDENT NUMBER: .....

TELEPHONE NUMBERS: .....

- Answer all questions.
- No calculators are allowed.
- Motivate all answers clearly.

1. Determine whether the following statements are TRUE or FALSE. In each case justify your answer.

   (a) Let W be a subspace of an inner product space. If u is a vector both in W and $W^{\perp}$, then $u = 0$. (2)

   (b) The matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 1 \\ 1 & 3 & 1 \end{bmatrix}$ has a QR-decomposition. (2)

   (c) Every set of linearly independent vectors in an inner product space is orthogonal. (2)

   (d) Let A be an $m \times n$ matrix. If $A^T A$ is invertible, then A is invertible. (2)

   (e) The matrix $\begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$ is orthogonal. (2)

   (f) If A is a square matrix, then both $A^T A$ and AA^T are othoganal diagonalizable. (2)

2. Formulate the Cauchy-Swartz inequality. (2)

3. Prove that if A is an othogonal matrix, then $|A| = \pm 1$. [Hint. Use the definition.] (2)

4. Find all complex scalars k, if any, for which u and v are othogonal if
$u = (3i, 1, i), \quad v = (-i, 5i, k)$. (5)

5. Let $A = \begin{bmatrix} 0 & 3+i \\ 3-i & -3 \end{bmatrix}$.

(a) Explain why A is Hermitian. (1)

(b) Find a unitary matrix P that diagonalizes A. (9)

(c) Calculate P^*AP. (2)

6. Consider the linear system

$$\begin{aligned} x_1 - 2x_2 &= 4 \\ 3x_1 - 6x_2 &= 1 \\ 2x_1 - 4x_2 &= 2. \end{aligned}$$

(a) Find all least square solutions of this system. (7)

(b) Calculate the error vector and the error. (3)

7. Consider the system

$$\begin{aligned} y_1' &= -4y_1 + y_2 \\ y_2' &= 2y_1 - 3y_2. \end{aligned}$$

(a) Solve this system. (12)

(b) Find the solution that satisfies the initial conditions $y_1(0) = 0, \quad y_2(0) = 0.$ (2)

8. Let A be a symmetric matrix. Explain what the spectral decomposition of A is. (3)