UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS

MODULE ASMA1B1
APPLICATIONS OF CALCULUS (ALTERNATIVE SEMESTER)

CAMPUS APK

EXAM JUNE EXAM 2016

DATE 31/05/2016 **SESSION** 12:30 – 14:30

ASSESSOR(S) DR A CRAIG
MR C HATANGIMANA
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INTERNAL MODERATOR MS S RICHARDSON

DURATION 2 HOURS **MARKS** 70

SURNAME AND INITIALS ______________________________

STUDENT NUMBER ______________________________

CONTACT NR ______________________________

NUMBER OF PAGES: 1 + 10 PAGES

INSTRUCTIONS:
1. ANSWER ALL THE QUESTIONS ON THE PAPER IN PEN.
2. NO CALCULATORS ARE ALLOWED.
3. SHOW ALL CALCULATIONS AND MOTIVATE ALL ANSWERS.
4. IF YOU REQUIRE EXTRA SPACE, CONTINUE ON THE <u>ADJACENT</u> BLANK PAGE NEXT TO IT AND INDICATE THIS CLEARLY.

Question 1 [3 marks]

Consider the function $f(x) = \dfrac{e^x}{x}$.

(a) Find the intervals of increase and decrease if it is given that $f'(x) = \dfrac{e^x(x-1)}{x^2}$. (2)

(b) Find the coordinates of the point(s) where any local maxima or minima occur. Indicate whether your point(s) are local maxima or minima. (1)

Question 2 [6 marks]

(a) State the Extreme Value Theorem. (2)

(b) Prove Rolle's Theorem. That is, prove the following: (4)

Let f be a function that satisfies the following hypothesis:

(1) f is continuous on the closed interval $[a, b]$;

(2) f is differentiable on the open interval (a, b);

(3) $f(a) = f(b)$.

Then there exists $c \in (a, b)$ such that $f'(c) = 0$.

Question 3 [2 marks]

Sketch the graph of a function that satisfies the following properties: (2)
$f'(x) < 0$ and $f''(x) < 0$ when $x < -1$
$f'(x) < 0$ and $f''(x) > 0$ when $-1 < x < 2$
$f'(x) > 0$ and $f''(x) > 0$ when $2 < x < 4$
$f'(x) > 0$ and $f''(x) > 0$ when $x > 4$

Question 4 [14 marks]

Evaluate the following integrals:

(a) $\displaystyle\int e^x \sin x \, dx$ (3)

(b) $\displaystyle\int \tan^3 x \sec^4 x \, dx$ (3)

(c) $\displaystyle\int \frac{dx}{x^2\sqrt{9-x^2}}$ (4)

(d) $\displaystyle\int \frac{x^2 - x + 6}{x^3 + 3x}\,dx$ (4)

Question 5 [3 marks]

Determine whether the following integral is divergent or convergent: $\displaystyle\int_1^e \frac{dx}{x\ln x}$ (3)

Question 6 [3 marks]

(a) Find the average value of $f(x) = \dfrac{1}{x}$ over the interval $[1, 3]$. (2)

(b) Find the value of c such that $f(c) = f_{ave}$. (1)

Question 7 [3 marks]

Set up an integral to find the length of the curve $y = \dfrac{x^2}{2} - \dfrac{\ln x}{4}$ for $2 \leqslant x \leqslant 4$. Simplify the integrand as far as possible but do not evaluate the integral. (3)

Question 8 [4 marks]

Sketch the region bounded by the given curves and calculate the area of the region. (4)

$$y = -x^2 + 4 \qquad y = 2x + 1$$

Question 9 [3 marks]

Use the **method of cylindrical shells** to set up an integral for the volume of the solid generated by rotating the region bounded by the following curves about the x-axis (do not evaluate the integral): (3)

$$y = 2\sqrt{x}, \qquad y = x - 3, \qquad y = 0$$

Question 10 [5 marks]

Solve the differential equations.

(a) $xy' + y = x^2 + 1$ with $y(1) = \dfrac{10}{3}$ (3)

(b) $\dfrac{dy}{dx} = \dfrac{2x + \sec^2 x}{2y}$ (2)

Question 11 [6 marks]

(a) Find the two real numbers a and b such that their sum is equal to 11, and such that they give a minimum value for $a^2 + b^2$. (3)

(b) At 11am a train starts travelling East at 45km/h. At 12pm, another train starts travelling South from the same point at 60km/h. How fast is the distance between them changing at 3pm? (3)

Question 12 [2 marks]

Show that the surface area of a sphere with radius r is $4\pi r^2$. (2)

Question 13 [2 marks]

Set up, but do not evaluate, an integral for the length of one arch of the cycloid: (2)

$$x = 2(t - \sin t),\ y = 2(1 - \cos t), \quad 0 \leqslant t \leqslant 2\pi$$

Question 14 [5 marks]

(a) Write the equation of the conic section given below in standard form, determine the focus, vertex and directrix, and sketch the graph. (3)

$$x^2 - 2x + 1 = 4y - 8$$

(b) Find the equation of the hyperbola with vertices $(0, \pm 2)$ and foci $(0, \pm 5)$. (2)

Question 15 [6 marks]

(a) Sketch the polar curves $r = 3\cos\theta$ and $r = 1 + \cos\theta$ on the same set of axes. Mark all intersections with the axes as well as the points where the curves intersect one another. (4)

(b) Set up, but do not evaluate, an integral for the area of the region inside $r = 1 + \cos\theta$. (2)

Question 16 [3 marks]

Use the Binomial Theorem to expand $(3x^2+6y^3)^5$. Calculate all combinations but do not calculate exponents. (3)