UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF PURE AND APPLIED MATHEMATICS

MODULE: **ASMA1A1**

COURSE: **CALCULUS OF ONE VARIABLE FUNCTIONS (ALTERNATIVE SEMESTER)**

CAMPUS: **APK**

EXAM: **NOVEMBER 2016**

DATE: **28/11/2016**

TIME: **16:30 – 18:30**

ASSESSOR: **MR SD NGIDI**

INTERNAL MODERATOR: **DR A CRAIG**

DURATION: **2 HOURS** **MARKS: 70**

SURNAME AND INITIALS

STUDENT NUMBER

CONTACT NUMBER

NUMBER OF PAGES: **1+12 PAGES (including front page)**

INSTRUCTIONS: **ANSWER ALL THE QUESTIONS IN PEN**

CALCULATORS ARE NOT ALLOWED.

If you require extra space, continue on the adjacent blank page next to it and indicate this clearly.

Question 1 [10 marks]

For questions 1.1 - 1.10, choose **one** correct answer, and make a cross (X) in the correct block.

Question	**a**	**b**	**c**	**d**	**e**
1.1					
1.2					
1.3					
1.4					
1.5					
1.6					
1.7					
1.8					
1.9					
1.10					

1.1 The rate of change of x with respect to t is written as: [1]

a) $\dfrac{dt}{dx}$

b) $\dfrac{d^2t}{dx^2}$

c) $\dfrac{d^2x}{dt^2}$

d) $\dfrac{dx}{dt}$

e) None of the above

1.2 $1 + \cot^2\theta =$ [1]

a) $\csc\theta$

b) $\sec\theta$

c) $\tan^2\theta$

d) $1 - \sin^2\theta$

e) None of the above

1.3 The expansion of $\displaystyle\sum_{i=3}^{5}(-1)^i\frac{2^i}{i}$ is: [1]

a) $-\frac{7}{4} + 3 - \frac{34}{6}$

b) $-\frac{8}{3} + 4 - \frac{32}{5}$

c) $\frac{9}{2} - 2 + \frac{32}{2}$

d) $4 - \frac{3}{8} + 4$

e) None of the above

1.4 If $f(x) = x^3 - 1$, $g(x) = \sqrt[3]{x^2 - 1}$ and $h(x) = \sqrt{x + 2}$, then $(f \circ g \circ h)(x)$ equals: [1]

a) 1

b) -1

c) 2

d) 3

e) None of the above

1.5 Solving $|x + 5| \geq 7$ yields [1]

a) $x \leq -12$ or $x \geq 2$

b) $-12 \leq x \leq 2$

c) $x \leq -10$ or $x \geq 4$

d) $x \leq 14$ or $x \geq -6$

e) None of the above

1.6 $\int (x^4 - x^{-1} + \sec^2 x)\, dx$ [1]

a) $\frac{1}{5}x^5 - 1 + \tan x + C$

b) $\frac{1}{5}x^5 - \ln|x| + \sec x + C$

c) $\frac{1}{5}x^5 - \ln|x| + \tan x + C$

d) $\frac{1}{5}x^5 - \ln|x| + \sec x \tan x + C$

e) None of the above

1.7 Find the limit: $\lim_{x \to 0} \left(\frac{\sin \pi}{\pi} \right)$ [1]

a) 1

b) 0

c) ∞

d) π

e) None of these

1.8 $\dfrac{d}{dx}\left[\cos^3(7^x)\right]$ [1]

a) $-\sin^3(7^x).\ln 7$

b) $3\cos^2(7^x).\sin(7^x)$

c) $-3\cos^2(7^x).\sin(7^x).\ln 7$

d) $-3\cos^2(7^x).\sin(7^x).(7^x)\ln 7$

e) None of these

1.9 Complete the following identity: $\cosh x - \sinh x =$ [1]

a) 1

b) -1

c) e^x

d) e^{-x}

e) None of these

1.10 If $\int_1^{10} f(x)\ dx = 20$, and $\int_8^{10} f(x)\ dx = 4$, then $\int_1^8 f(x)\ dx =$ [1]

a) 24

b) 16

c) 5

d) -16

e) None of these

Question 2 [7 marks]

a) Rewrite the first-order statement in natural language $\forall x \in \mathbb{R}, x^2 \geq 0 \rightarrow x > 0$ [1]

b) Write the negation of the sentence given in 2 a) using natural language. [2]

c) Use logical equivalences to rewrite the statement in 'If then' form using natural language: "Sandile wakes up early or he misses the bus to school" [2]

d) Let p and q be true propositions and let r be false proposition.
Determine the truth value of:

$$\neg(\neg(\neg r \wedge (p \rightarrow \neg q)))$$

[2]

Question 3 [11 marks]

a) Solve for x:

$$\frac{x}{x^2+2x-3} < -\frac{2}{x^2+2x-3}$$

[4]

b) If f is the function defined below, determine whether f is even, odd or neither:

$$f(x) = 2 - 3\cos x$$

[2]

c) Determine the inverse of $f(x) = 3\ln(1 - e^x)$. [3]

d) Sketch the graph of $\csc\theta$ for $0 < \theta < \pi$. Indicate the co-ordinates of any turning points and label the asymptotes. [2]

Question 4 [2 marks]

Let $f(x) = 2\ln(x-1) + 1$ find:

a) the domain of $f(x)$. [1]

b) the range of $f(x)$ [1]

Question 5 [4 marks]

Determine:

a) $\lim_{x \to -2} \dfrac{x^4 - 16}{x + 2}$ [2]

b) $\lim_{x \to -\infty} \dfrac{\sqrt{4x^6 - x}}{x^3 + 5}$ [2]

Question 6 [6 marks]

a) Sketch

$$f(x) = \begin{cases} e^{-x} & \text{if} \quad x < 0 \\ \tan x & \text{if} \quad x \geq 0 \end{cases}$$

[3]

b) Is $f(x)$ continuous at $x = 0$? Explain. [1]

c) Is $f(x)$ differentiable at $x = 0$? Explain. [1]

d) Explain why the function

$$f(x) = \ln|x - 2|$$

is discontinuous at $x = 2$. [1]

Question 7 [3 marks]

If $\tan\theta = -\sqrt{3}$ and $\pi < \theta < 2\pi$, find the five other trigonometric ratios. [3]

Question 8 [3 marks]

If $f(x) = \dfrac{1}{3\sqrt{x}}$ determine $f'(x)$ by making use of first principles. [3]

Question 9 [3 marks]

Find the equation of the tangent line to the curve at the given point

$$y = \frac{4x}{x^2+3}, \quad x = 3$$

[3]

Question 10 [7 marks]

Differentiate the following functions:

a) $f(x) = \sin(4x^3 + 3x + 1)$ [1]

b) $e^{xy} + 2y - 3x = \sin y$ [3]

c) By using logarithmic differentiation: $y = \left(1 + \frac{1}{x}\right)^x$ [3]

Question 11 [5 marks]

a) State the Fundamental Theorem of Calculus part 1. [2]

b) Given

$$f(x) = \int_1^{\sqrt{x}} \frac{2t^2}{t^4 + 1}\, dt$$

evaluate $f'(x)$ by using part 1 of the Fundamental Theorem of Calculus. [3]

Question 12 [5 marks]

Evaluate the following integrals

a) $\int_1^4 \left(\sqrt{x} - \frac{1}{x^2}\right)\, dx$ [2]

b) $\int x\cos(x^2)\, dx$ [3]

Question 13 [4 marks]

Prove the product rule:
If f and g are both differentiable, then

$$\frac{d}{dx}\left[f(x)g(x)\right] = f(x)\frac{d}{dx}\left[g(x)\right] + g(x)\frac{d}{dx}\left[f(x)\right]$$

[4]